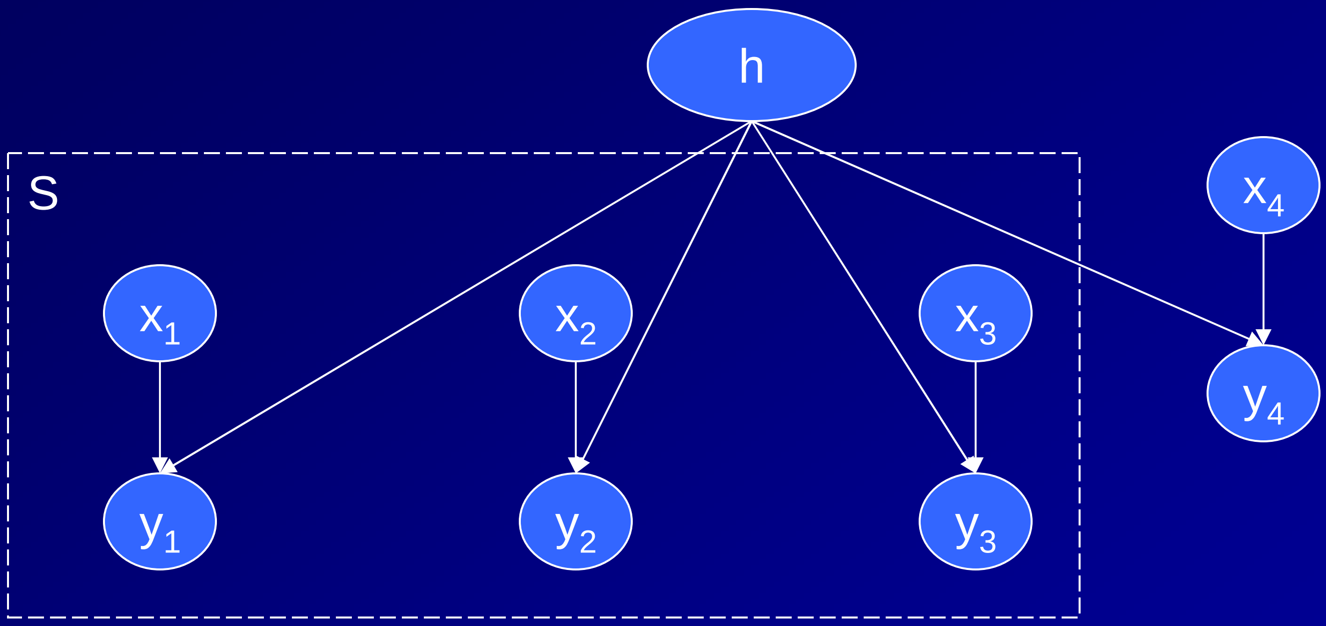
Bayes Network description of the learning problem



$$P(S, h, x_4, y_4) = \prod_i P(y_i \mid x_i, h)\, P(x_i)\, P(h)\, P(y_4 \mid x_4, h)\, P(x_4)$$

$$= P(S|h)\, P(h)\, P(y_4 \mid x_4, h)\, P(x_4)$$

$$= P(h|S)\, P(S)\, P(y_4 \mid x_4, h)\, P(x_4)$$

Making a Prediction: Bayesian Model Averaging

- Goal: given S, x_4, predict y_4

$$
\begin{aligned}
P(y_4|x_4, S) &= \frac{P(S, x_4, y_4)}{P(S, x_4)} \\
&= \frac{\sum_h P(S, x_4, y_4, h)}{P(S)P(x_4)} \\
&= \frac{\sum_h P(h|S)P(S)P(x_4)P(y_4|x_4, h)}{P(S)P(x_4)} \\
&= \sum_h P(h|S)P(y_4|x_4, h)
\end{aligned}
$$

Maximum A Posteriori (MAP) Estimation

- Bayesian model averaging is usually infeasible to compute
- Replace the Bayesian model average by the best single model h^{MAP}

$$
\begin{aligned}
P(y=k|S,\mathbf{x}) &= \sum_{h \in H} P(y=k|h,\mathbf{x})P(h|S) \\
&\approx P(y=k|h^{MAP},\mathbf{x})
\end{aligned}
$$

where

$$h^{MAP} = \mathsf{argmax}_h P(h|S) = \mathsf{argmax}_h P(S|h)P(h)$$

MAP = Penalized Maximum Likelihood

- We can view P(h) as a "complexity" penalty on the maximum likelihood hypothesis

$$\begin{aligned} h^{MAP} &= \mathrm{argmax}_h P(S|h)P(h) \\ &= \mathrm{argmax}_h \log P(S|h) + \log P(h) \\ &= \mathrm{argmax}_h \, \ell(h) + \log P(h) \end{aligned}$$

Where does P(H) come from?

- Theory: P(H) should encode *all and only* our prior knowledge about H.
- Practice:
  - Complexity-based priors
    - penalize large neural network weights
    - penalize large SVM weights
    - penalize large decision trees
    - penalize long “description lengths”
  - Knowledge-based priors
    - Bayes net structure prior
    - qualitative monotonicity priors
    - smoothness priors