BLIND SPECTRA SEPARATION AND DIRECTION FINDING FOR COGNITIVE RADIO USING TEMPORAL CORRELATION-DOMAIN ESPRIT

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ABSTRACT

Unraveling power spectra mixtures and finding the directions of the constituent sources can enable effective spatial occupancy prediction by location-dependent combining of the recovered source power spectra; and it is also useful for primary interference avoidance. Such unmixing and direction-finding is a challenging leap beyond ordinary 'aggregate' spectrum sensing. This paper presents a promising new method for blind (power) spectra separation and emitter direction finding using a network of cognitive radios. Each radio has a pair of antennas, and the baselines of different radios are aligned (e.g., using a compass), in a configuration *reminiscent* of classical *spatial* correlation-based ESPRIT. Unlike classical ESPRIT, array geometry is exploited here in the *temporal* correlation domain to come up with a simple and effective blind spectra separation and direction finding solution with guaranteed identifiability and robustness to noise. A notable feature is that the different radios need not be synchronized, as they do in spatial ESPRIT.

***Index Terms*—** Blind Power Spectra Separation, Direction-of-Arrival Estimation, Spectrum Sensing, Cognitive Radio, ESPRIT

1. INTRODUCTION

Cognitive radio can help resolve the problem of spectrum scarcity, by judiciously exploring under-utilized frequency bands and exploiting transmission opportunities, while respecting the licensed primary users. *Spectrum sensing* is the first step towards this end, as it enables the situational awareness needed for intelligent spectrum reuse.

The spectrum sensing task has been formulated in various ways. Most references to date treat the task as binary hypothesis testing for each frequency bin; see [1] for a recent comprehensive survey of this type of approaches. Broadband sensing has also been considered, in an attempt to exploit pertinent sparsity and correlation properties, e.g., [2–4]. Here we consider taking the sensing task to the next step, from aggregate power spectrum sensing to power spectra separation and localization of spectrally overlapping transmissions from multiple sources. Knowing the individual power spectrum and the direction of each source is useful for a number of reasons. For instance, combining the power spectra 'atoms' with direction information allows spatial power spectrum interpolation, as well as limiting interference to licensed users - in addition to security, integrity, and signal intelligence implications. The power spectra separation and direction finding problem is also challenging - which of course just adds to its research appeal. Specifically, as no cooperation or control signaling between primary and cognitive radio (CR) systems can be assumed, the separation process has to be implemented blindly at the CR sensors.

A recent article [5] has formulated a variation of the aforementioned problem, assuming a pure path loss model without shadowing or fading, as non-negative matrix factorization (NMF). NMF does not necessarily yield the true underlying power spectra, as the solution to NMF is not guaranteed to be unique. Furthermore, NMF is affected by measurement noise, and its solution entails significant computational complexity.

In this paper, our primary goal is to provide a theoretically sound formulation, as well as a simple and practically implementable solution to the joint blind spectra separation and emitter direction finding problem for cognitive radio applications. Our solution employs a network of cognitive radios. Each radio has a pair of antennas, and the baselines of different radios are aligned (e.g., using a compass), in a configuration that is similar to classical *spatial* correlation-based ESPRIT, as used for Direction Of Arrival (DOA) estimation. Unlike classical ESPRIT, we do not need synchronization across the big virtual subarray, i.e., the two down-conversion chains corresponding to the receive antennas of each cognitive radio should be synchronized, but different radios need not be synchronized. Instead of working with the spatial correlation of the virtual subarray, here the special array geometry is exploited in the *temporal* correlation domain to come up with a simple and effective blind spectra separation and direction finding solution with guaranteed identifiability and robustness to noise. As ESPRIT reduces to an eigenvalue problem, it avoids overly complicated computations. Together with the ability to work without network-wide synchronization, this renders the overall solution practically feasible in commodity software radios, for example. Finally, the method includes a projection step that mitigates the influence of additive white measurement noise, thus yielding more accurate estimation results. Simulations illustrate the effectiveness of the approach for both blind power spectra separation and emitter DOA estimation.

2. SETUP AND SIGNAL MODEL

Consider a scenario where there are K primary or secondary transmitters in the far field of a network of N cognitive radio receivers, each equipped with two receive antennas and down-conversion chains. Let $x_k(t) \in \mathbb{C}$, for $t = 0, 1, \ldots$, denote the k-th transmitted signal. We assume that $x_k(t)$ is wide-sense stationary, and its bandwidth is relatively narrow compared to its carrier frequency (e.g., 20 MHz in the 2-5 GHz band; otherwise we may apply our approach to sub-bands). Without loss of generality, we also assume that $\mathbb{E}\{|x_k(t)|^2\} = 1$, for $k = 1, \ldots, K$. The two received signals

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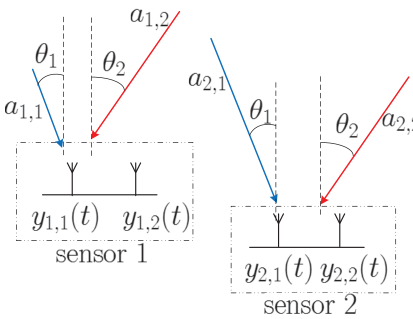

Fig. 1: The considered scenario when $(N, K) = (2, 2)$.

at cognitive radio n can be expressed as

$$\boldsymbol{y}_n(t) = \boldsymbol{\Phi}_n \boldsymbol{A}_n \boldsymbol{x}(t) + \boldsymbol{\upsilon}_n(t), \quad t = 0, 1, 2, ..., \tag{1}$$

where $\boldsymbol{x}(t) = [\, x_1(t), \ldots, x_K(t) \,]^T$ denotes the vector of transmitted signals, $\boldsymbol{y}_n(t) = [\, y_{n,1}(t), y_{n,2}(t) \,]^T$ denotes the two received signals at the antennas of cognitive radio n, $\boldsymbol{\upsilon}_n(t) = [\, \upsilon_{n,1}(t), \upsilon_{n,2}(t) \,]^T$ denotes the corresponding noise vector, $\boldsymbol{A}_n = \text{Diag}(\, a_{n,1}, \ldots, a_{n,K} \,)$ is a diagonal matrix whose kth diagonal element $a_{n,k}$ contains the channel response from transmitter k to receiver n, and $\boldsymbol{\Phi}_n$ is a matrix which is generally related to the geometry of the antenna placement and the DOA of the received wavefronts at sensor n. Here we consider the case where each cognitive radio receiver employs a pair of dipoles or monopoles, with fixed common spacing. The Universal Software Radio Peripheral (USRP) by Ettus Research is a popular software radio platform that fits this bill, for example. Consequently, we have [6]

$$\boldsymbol{\Phi}_n = \begin{bmatrix} 1, & \ldots, & 1 \\ e^{j\phi_{n,1}}, & \ldots, & e^{j\phi_{n,K}} \end{bmatrix},$$

where $\phi_{n,k} = -2\pi d \sin(\theta_{n,k})/\lambda$, $\theta_{n,k} \in [\, -\pi, \pi \,]$ is the DOA of source k at sensor n, λ is the wavelength corresponding to the carrier frequency, and d denotes the distance between the two receive antennas. We further assume that the baselines of the receiving radios are aligned (e.g., using a compass) to face in the same direction, such that

$$\theta_{n,k} = \theta_k, \; k = 1, \ldots, K,$$

which leads to $\phi_{n,k} = \phi_k$. The scenario we just described is illustrated in Fig. 1. Assuming that $\{\boldsymbol{y}_n(t)\}_{n=1,\ldots,N}$ and (a perhaps tentative estimate of) K are available, our interest lies in blindly separating the K underlying power spectra and estimating $\{\theta_1, \ldots, \theta_K\}$. It is worth noting from early on that our solution will not require synchronization between the N different receiving radios, and it will not involve communicating the raw received signals to the fusion center - only reduced temporal auto- and cross-correlation summaries need to be communicated, which is of course advantageous from a control signaling overhead point of view.

3. PROPOSED APPROACH

In this section, we propose a problem formulation following the described signal model. As we will see, our formulation makes use of the signal structure induced by the special antenna geometry, and leads to an algebraically simple solution to the simultaneous blind power spectra separation and DOA estimation problem.

3.1. Problem Formulation

To get started, let us define

$$c_{n,q}(\ell) = \mathbb{E}\left\{ y_{n,1}(t) y_{n,q}^*(t-\ell) \right\}, \quad q = 1, 2. \tag{2}$$

In words, $c_{n,q}(\ell)$ denotes the ℓ-lag auto/cross-correlation between the received signals at the first and the qth antennas of cognitive radio receiver (henceforth called *sensor* for brevity) n. Notice that by our signal model, we have $y_{n,q}(t) = \sum_{k=1}^K a_{n,k} e^{j(q-1)\phi_k} x_k(t) + \upsilon_{n,q}(t)$ for $q = 1, 2$. Hence, by assuming that the the transmitted signals are (naturally) mutually uncorrelated, and that the noises are also uncorrelated to the source signals, one can see that for $q = 1, 2$,

$$c_{n,q}(\ell) = \sum_{k=1}^K |a_{n,k}|^2 e^{j(1-q)\phi_k} r_k(\ell) + \mathbb{E}\{\upsilon_{n,1}(t)\upsilon_{n,q}^*(t-\ell)\},$$

where $r_k(\ell) = \mathbb{E}\{x_k(t) x_k^*(t-\ell)\}$ is the autocorrelation of $x_k(t)$ at time lag ℓ. Assume that the noise $\upsilon_{n,q}(t)$ at each antenna of sensor n is white Gaussian, both temporally and spatially, with zero mean and variance σ_n^2, i.e., $\upsilon_{n,q}(t) \sim \mathcal{N}(0, \sigma_n^2)$ for $q = 1, 2$. Then, when $q = 1$, we have

$$\mathbb{E}\{\upsilon_{n,1}(t)\upsilon_{n,1}^*(t-\ell)\} = \sigma_n^2 \delta(\ell), \tag{3}$$

where $\delta(\ell)$ denotes the Kronecker delta

$$\delta(\ell) = \begin{cases} 1, & \ell = 0, \\ 0, & \text{otherwise}. \end{cases} \tag{4}$$

When $q = 2$, we have

$$\mathbb{E}\{\upsilon_{n,1}(t)\upsilon_{n,q}^*(t-\ell)\} = 0, \; \forall \ell, \tag{5}$$

as the noises are uncorrelated to each other. Thus, we can compactly express $c_{n,q}(\ell)$ as

$$c_{n,q}(\ell) = \begin{cases} \sum_{k=1}^K |a_{n,k}|^2 r_k(\ell) + \sigma_n^2 \delta(\ell), & q = 1, \\ \sum_{k=1}^K |a_{n,k}|^2 e^{-j\phi_k} r_k(\ell), & q = 2. \end{cases} \tag{6}$$

Now, consider the discrete-time Fourier transform (DTFT) of $\{c_{n,q}(\ell)\}_{\ell=-\infty}^{\infty}$. By the linearity of DTFT and the basic transform pair $\delta(\ell) \leftrightarrow 1$, we have

$$\begin{aligned} C_{n,q}(\omega) &= \sum_{\ell=-\infty}^{\infty} c_{n,q}(\ell) e^{-j\omega\ell} \\ &= \begin{cases} \sum_{k=1}^K |a_{n,k}|^2 S_k(\omega) + \sigma_n^2, & q = 1, \\ \sum_{k=1}^K |a_{n,k}|^2 e^{-j\phi_k} S_k(\omega), & q = 2. \end{cases} \end{aligned} \tag{7}$$

where $\omega \in [-\pi, \pi]$, and $S_k(\omega) = \sum_{\ell=-\infty}^{\infty} r_k(\ell) e^{-j\omega\ell}$ denotes the power spectrum of source k at frequency ω. We discretize the frequency axis to N_F samples and denote

$$G_{n,q}(f) = C_{n,q}\left(\frac{2\pi f}{N_F}\right),$$

where $f = 0, 1, \ldots, N_F - 1$. Notice that in practice such $G_{n,q}(f)$ can be computed by applying the discrete Fourier transform (DFT) to $\{c_{n,q}(\ell)\}_{\ell \in \mathcal{L}}$, where $\mathcal{L}$ denotes the index set of available time lags. Given the obtained $G_{n,q}(f)$, we construct $\boldsymbol{G}_q \in \mathbb{R}^{N \times N_F}$ for $q = 1, 2$ such that

$$[\boldsymbol{G}_q]_{n,f+1} = G_{n,q}(f), \quad f = 0, 1, \ldots, N_F - 1. \tag{8}$$

We can compactly express $\boldsymbol{G}_q$ as

$$\boldsymbol{G}_1 = \boldsymbol{B}\boldsymbol{S}^T + \boldsymbol{\eta}\mathbf{1}^T, \tag{9a}$$

$$\boldsymbol{G}_2 = \boldsymbol{B}\boldsymbol{D}\boldsymbol{S}^T, \tag{9b}$$

where $\boldsymbol{B} = \boldsymbol{A} \circ \boldsymbol{A}^*$, "$\circ$" denotes the Hadamard product, $\boldsymbol{A} \in \mathbb{C}^{N \times K}$ such that $[\boldsymbol{A}]_{n,k} = a_{n,k}$, $\boldsymbol{D} = \mathrm{Diag}(e^{-j\phi_1}, \ldots, e^{-j\phi_K})$ contains the desired DOA parameters in its diagonal elements, $\boldsymbol{S} = [\ \boldsymbol{s}_1, \ldots, \boldsymbol{s}_K\ ]$, $\boldsymbol{s}_k = [S_k(0), \ldots, S_k(N_F - 1)]^T$ denotes the discretized power spectrum of transmitter k, which we also aim to estimate, $\mathbf{1}$ is an all-one vector with proper length, and $\boldsymbol{\eta} = [\sigma_1^2, \ldots, \sigma_N^2]^T$ contains all the sensor noise variances.

To get rid of the noise term in (9a), let $\boldsymbol{P}^\perp = \boldsymbol{I} - (1/N_F)\mathbf{1}\mathbf{1}^T$ be the orthogonal complement projector of $\mathbf{1}$ and obtain

$$\tilde{\boldsymbol{G}}_q = \boldsymbol{G}_q\boldsymbol{P}^\perp, \quad q = 1, 2.$$

Then, for $q = 1$ we will subsequently have $\boldsymbol{G}_1\boldsymbol{P}^\perp = \boldsymbol{B}\boldsymbol{S}^T\boldsymbol{P}^\perp + \boldsymbol{\eta}(\boldsymbol{P}^\perp\mathbf{1})^T$, where the second term is zero (when working with exact correlation matrices). Hence, at this point, our problem boils down to using $\tilde{\boldsymbol{G}}_1 = \boldsymbol{B}\boldsymbol{S}^T\boldsymbol{P}^\perp$ and $\tilde{\boldsymbol{G}}_2 = \boldsymbol{B}\boldsymbol{D}\boldsymbol{S}^T\boldsymbol{P}^\perp$ to estimate $\boldsymbol{S}$ and $\theta_1, \ldots, \theta_K$.

3.2. Solution via Rotational Invariance

We propose to employ the *estimation of signal parameters via rotational invariance technique* (ESPRIT) to estimate $\boldsymbol{D}$ and $\boldsymbol{S}$. ESPRIT is appealing in our context due to its ability to identify the sought DOA parameters and to separate the atomic power spectra, but also for simplicity and suitability for practical implementation. For identifiability, we assume that

(A1) $\mathrm{rank}(\boldsymbol{B}) = K$.
(A2) $\theta_1, \ldots, \theta_K$ are distinct.
(A3) $\mathrm{rank}\left([\boldsymbol{S}, \mathbf{1}]\right) = K + 1$.

(A1) implies $N \geq K$, so that the number of (dual-channel) receivers should be larger than or equal to the number of active transmitters. For $N \geq K$, $\mathrm{rank}(\mathbf{B}) = K$ almost surely if $\boldsymbol{A}$ is drawn from a jointly continuous distribution, such as, for example, when $\boldsymbol{A}$ is modeled as i.i.d. circularly symmetric complex Gaussian, in which case $\boldsymbol{B}$ is i.i.d. Chi-square distributed [7, 8]. (A3) says that the underlying atomic power spectra are all linearly independent, and they cannot be linearly combined to yield a white noise-like frequency-flat power spectrum (because then white measurement noise cannot be separated from the sought spectra).

For a quick retrospective of ESPRIT, we first construct $\tilde{\boldsymbol{G}} = [\tilde{\boldsymbol{G}}_1^T, \tilde{\boldsymbol{G}}_2^T]^T$. Then, let $\boldsymbol{U} \in \mathbb{C}^{2N \times K}$ denote the first K left singular vectors of $\tilde{\boldsymbol{G}}$. Under (A1)-(A3), it is not difficult to see that $\mathrm{rank}(\boldsymbol{D}) = \mathrm{rank}(\boldsymbol{S}^T\boldsymbol{P}^\perp) = K$ and thus

$$\mathcal{R}(\boldsymbol{U}) = \mathcal{R}\left(\begin{bmatrix}\boldsymbol{B}\\ \boldsymbol{B}\boldsymbol{D}\end{bmatrix}\right). \tag{10}$$

Let $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ contain the first and second N rows of $\boldsymbol{U}$, respectively. One can see from Eq. (10) that there exists a full-rank square matrix $\boldsymbol{Q} \in \mathbb{C}^{K \times K}$ such that

$$\boldsymbol{U}_1 = \boldsymbol{B}\boldsymbol{Q}, \tag{11a}$$

$$\boldsymbol{U}_2 = \boldsymbol{U}_1\boldsymbol{\Psi}, \tag{11b}$$

$$\boldsymbol{\Psi} = \boldsymbol{Q}^{-1}\boldsymbol{D}\boldsymbol{Q}. \tag{11c}$$

Solving (11b) for $\boldsymbol{\Psi}$ in the least squares sense[1] yields

$$\hat{\boldsymbol{\Psi}} = \boldsymbol{U}_1^\dagger\boldsymbol{U}_2.$$

From (11c), the estimated $\hat{\boldsymbol{D}}$ and $\hat{\boldsymbol{Q}}$ can be obtained by eigen-decomposition of $\hat{\boldsymbol{\Psi}}$, which is unique under (A2). Thus $\hat{\boldsymbol{B}}$ can be subsequently estimated. After obtaining $\hat{\boldsymbol{B}}$ and $\hat{\boldsymbol{D}}$, $\hat{\boldsymbol{S}}$ can be easily found using (9b), i.e.,

$$\hat{\boldsymbol{S}} = \left(\hat{\boldsymbol{D}}^{-1}\hat{\boldsymbol{B}}^\dagger\boldsymbol{G}_2\right)^T,$$

as $\boldsymbol{G}_2$ is noise-free in theory.

Remark 1 We would like to emphasize that the proposed approach does not require synchronization between the different radio receivers; only the two down-conversion chains of each individual receiver are assumed to be synchronized. This stems from the fact that the rows of $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$ are only related to the auto/cross-correlations of pairs of signals received at the same radio. We will in fact demonstrate that this is a key benefit of our temporal correlation-domain ESPRIT approach, relative to using spatial correlation-domain ESPRIT, which hinges upon accurate network-wide synchronization - which is very hard, if at all possible to maintain in practice.

Remark 2 By noticing that both $\boldsymbol{B}$ and $\boldsymbol{S}$ are non-negative, a simple NMF criterion can be considered for blind separation of the source spectra, i.e.,

$$\hat{\boldsymbol{S}} = \arg\min_{\boldsymbol{S}\geq\mathbf{0},\boldsymbol{B}\geq\mathbf{0}} \left\|\boldsymbol{G}_1 - \boldsymbol{B}\boldsymbol{S}^T\right\|_F^2, \tag{12}$$

where the inequalities are element-wise. A similar NMF idea was first proposed in [5] to localize the sources and estimate their power spectra simultaneously, albeit localization then hinges on a restrictive path loss-only model for $\boldsymbol{B}$ with a known exponent, which is applicable in line-of-sight situations. The advantage of this formulation is that each receiver needs to use only one antenna and associated down-conversion chain. The drawbacks are that NMF is not unique in general [9, 10], so localization and spectra separation may fail to work; and the computational complexity of NMF (an NP-hard problem) is much higher than that of eigen-decomposition-based ESPRIT. For our proposed approach, on the other hand, due to uniqueness of the ESPRIT solution, the power spectra are guaranteed to be recovered under our working assumptions if the $\boldsymbol{G}_q$'s are perfectly estimated.

4. SIMULATIONS

In this section, we simulate a scenario with $(N, K) = (3, 2)$. In each trial of our simulation, the transmitted signals are generated as passband-filtered circularly symmetric white Gaussian processes. Specifically, we partition the considered frequency band into twelve channels and randomly choose four channels for parallel transmission by each transmitter (the spectra of the two transmitters overlap with positive probability); in each active channel, the transmitted power spectrum is sinc-shaped with a random positive scaling factor. Such a setup is used to simulate sparse frequency band usage by the primary users. The two transmitted wavefronts are set to arrive at the receiver antenna doublets from directions $\theta_1 = -18^\circ$ and $\theta_2 = 5^\circ$. In each trial, the entries of the channel

[1] Total least squares may be used instead.

matrix $\boldsymbol{A}$ are drawn from the i.i.d. circularly symmetric complex Gaussian distribution. We assume that the noise variances at all receiving antennas are identical, i.e., $\sigma_1^2 = \ldots = \sigma_N^2 = \sigma^2$. The signal-to-noise ratio (SNR) is subsequently defined as $\text{SNR} = \mathbb{E}\left\{\|\boldsymbol{A}\boldsymbol{x}(t)\|_2^2/\sigma^2\right\}$. In all the simulations, $N_F = 256$ frequency bins and $\mathcal{L} = \{-255, -254, \ldots, 254, 255\}$ are used.

Fig. 2 shows the separated atomic power spectra for one randomly picked instance with sample size $T = 2 \times 10^5$ and SNR= 0dB. As a baseline for comparison, we also present the result using the NMF criterion in the last section; the multiplicative update algorithm [11] is employed to tackle the optimization problem in (12). One can see that even in this low SNR scenario, the proposed approach can well separate the constituent spectra, and gives a clear indication of the usage of the considered band. On the other hand, one can see that NMF is not as promising as the proposed method in terms of estimating the underlying power spectra in this scenario.

In Fig. 3, the mean-square-errors (MSEs) of the estimated power spectra $\hat{\boldsymbol{S}}$ obtained by NMF and the proposed temporal correlation-domain ESPRIT algorithms are presented; each result is averaged over 100 trials. One can see that for both $T = 5 \times 10^4$ and $T = 2 \times 10^5$ cases, the proposed approach yields lower MSEs than NMF. Also, with larger sample size, the proposed approach exhibits a lower MSE floor. Particularly, when $T = 2 \times 10^5$, the proposed approach exhibits more than 10dB lower MSE than that of NMF when SNR $\geq$ 0 dB. This can be explained by the fact that larger sample size results in more accurate estimation of $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$, which leads to better estimation of $\boldsymbol{S}$.

Fig. 4 shows root mean-square-error (RMSE) of the DOAs estimated using the proposed temporal correlation-domain ESPRIT approach, and the classical spatial correlation-domain ESPRIT approach (e.g., see [12]). One can verify that, if the sensors are synchronized, the spatial correlation-domain ESPRIT algorithm exhibits excellent RMSE performance, while the proposed temporal correlation-domain one works reasonably well, but it cannot compete with the classical method in terms of DOA estimation accuracy. The situation is reversed, and the proposed method shines exactly when there is a random timing mismatch between the different radio receivers, in which case classical spatial covariance-domain ESPRIT fails completely, whereas our method is virtually unaffected, and maintains RMSE lower than 0.1 degree when SNR $\geq$ 20 dB. This corroborates our earlier claim in Remark 1.

5. CONCLUSION

In this paper, we have proposed an approach for blindly separating the power spectra of different co-channel transmitters and estimating the corresponding DOAs using a network of dual-antenna cognitive radios with aligned baselines. The temporal auto- and cross-correlations of pairs of signals at each cognitive radio receiver are used together with this special receiving array geometry to formulate temporal correlation-domain ESPRIT problem that yields the underlying atomic power spectra and the DOAs of the active transmitters. A notable feature is that the different radios need not be synchronized, as they do in spatial ESPRIT. The overall solution is simple and it provides identifiability guarantees, unlike related formulations based on NMF. The results are useful for spatial occupancy prediction by location-dependent combining of the recovered source power spectra; primary interference avoidance; and secondary network integrity, security, and signal intelligence applications. Pertinent extensions of these ideas will be presented in a forthcoming journal version.

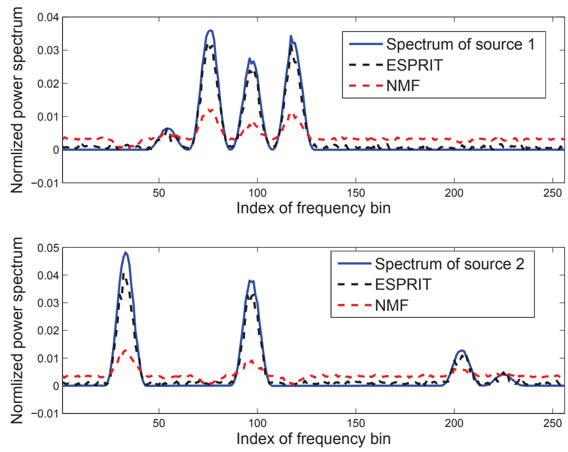

Fig. 2: The true and estimated PSDs of the Sources by the algorithms; sample size $T = 2 \times 10^5$; SNR= 0dB.

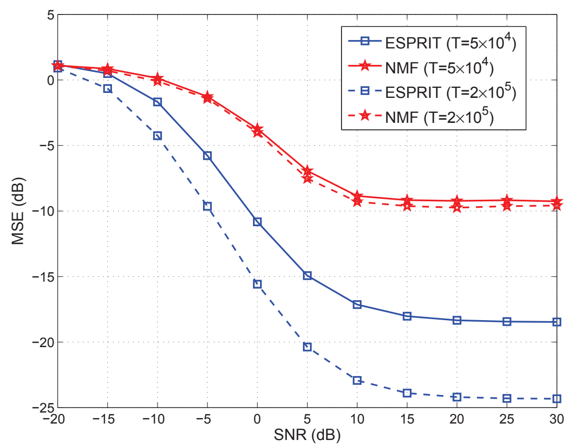

Fig. 3: The MSEs of the estimated spectra using ESPRIT and NMF under different SNRs.

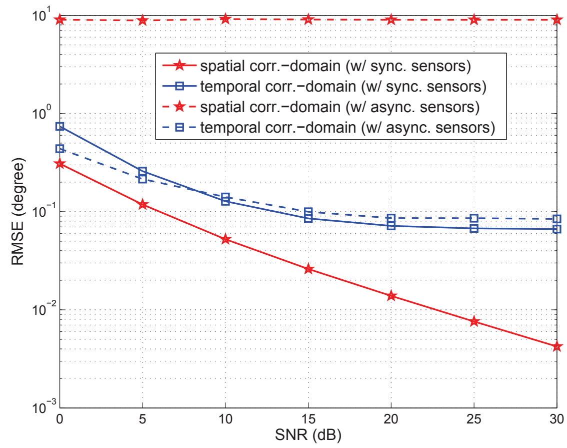

Fig. 4: The RMSEs of the estimated DOAs by ESPRIT in temporal correlation-domain and spatial correlation-domain, respectively; sample size $T = 2 \times 10^5$.

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