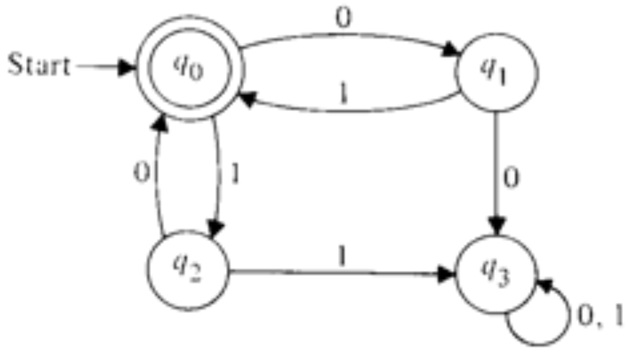
Language Technology

CUNY Graduate Center, Spring 2013

Unit 1: Sequence Models

Lecture 2: Finite-State Acceptors/Transducers



Liang Huang

This Week: Finite-State Machines

- Finite-State Acceptors and Languages
  - DFAs (deterministic)
  - NFAs (non-deterministic)
- Finite-State Transducers
- Applications in Language Processing
  - part-of-speech tagging, morphology, text-to-sound
  - word alignment (machine translation)
- Next Week: putting probabilities into FSMs

Languages and Machines

- Q1: how to formally define a *language*?
- a language is a set of strings
  - could be finite, but often infinite (due to recursion)
  - L = { aa, ab, ac, ..., ba, bb, ..., zz }   (finite)
  - English is the set of *grammatical English sentences*
  - variable names in C is set of alphanumeric strings
- Q2: how to *describe* a (possibly infinite) language?
  - use a finite (but recursive) representation
  - finite-state acceptors (FSAs) or regular-expressions

English Adjective Morphology

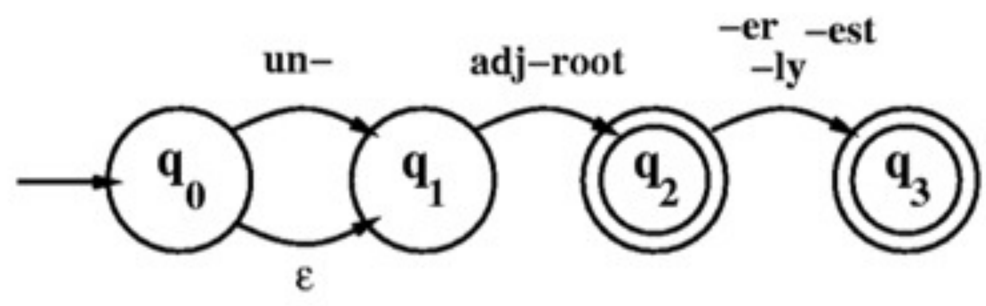


Figure 3.4 An FSA for a fragment of English adjective morphology: Antworth's Proposal #1.

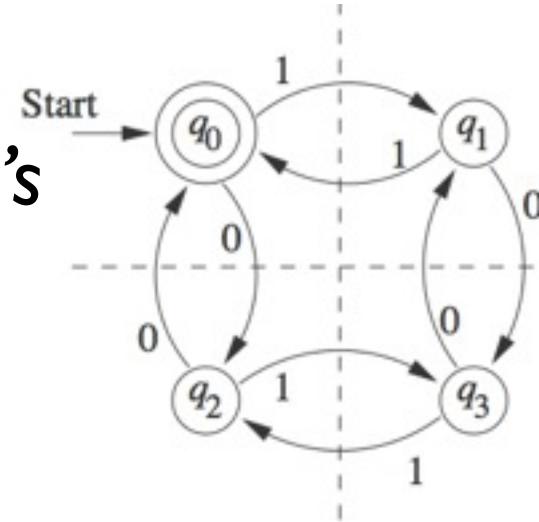
exceptions?

Finite-State Acceptors

- L1 = { aa, ab, ac, ..., ba, bb, ..., zz } (finite)
  - start state, final states
- L2 = { *all letter sequences* } (infinite)
  - recursion (cycle)
- L3 = { *all alphanumeric strings* }

More Examples

- L4 = { *all letter strings with at least a vowel* }
- L5 = { *all letter strings with vowels in order* }
- L6 = { *all 01 strings with even number of 0's and even number of 1's* }



English Adjective Morphology

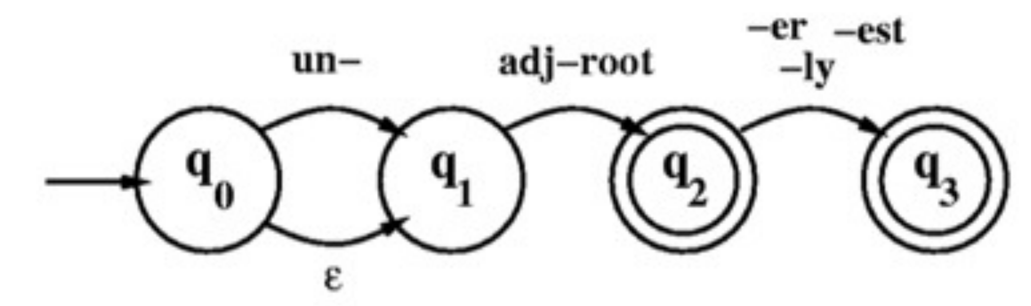


Figure 3.4 An FSA for a fragment of English adjective morphology: Antworth's Proposal #1.

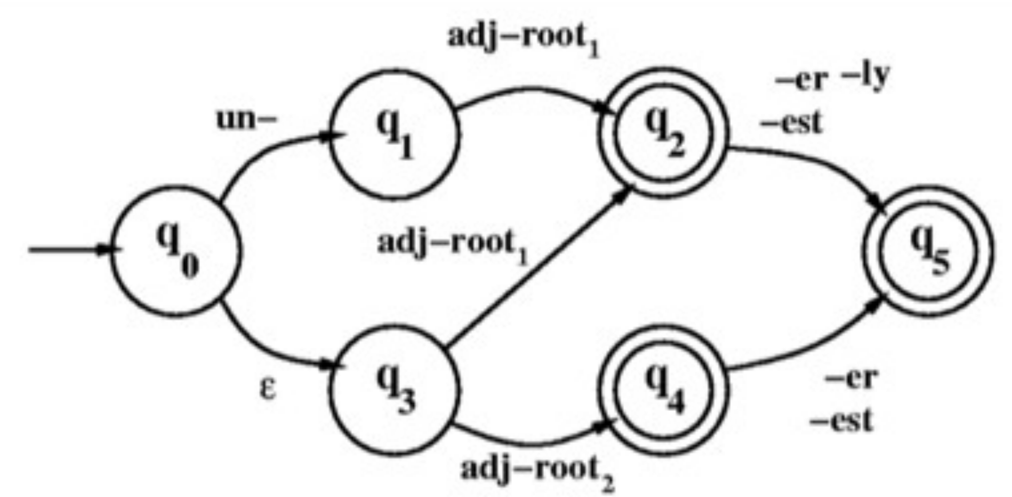


Figure 3.5 An FSA for a fragment of English adjective morphology: Antworth's Proposal #2.

More English Morphology

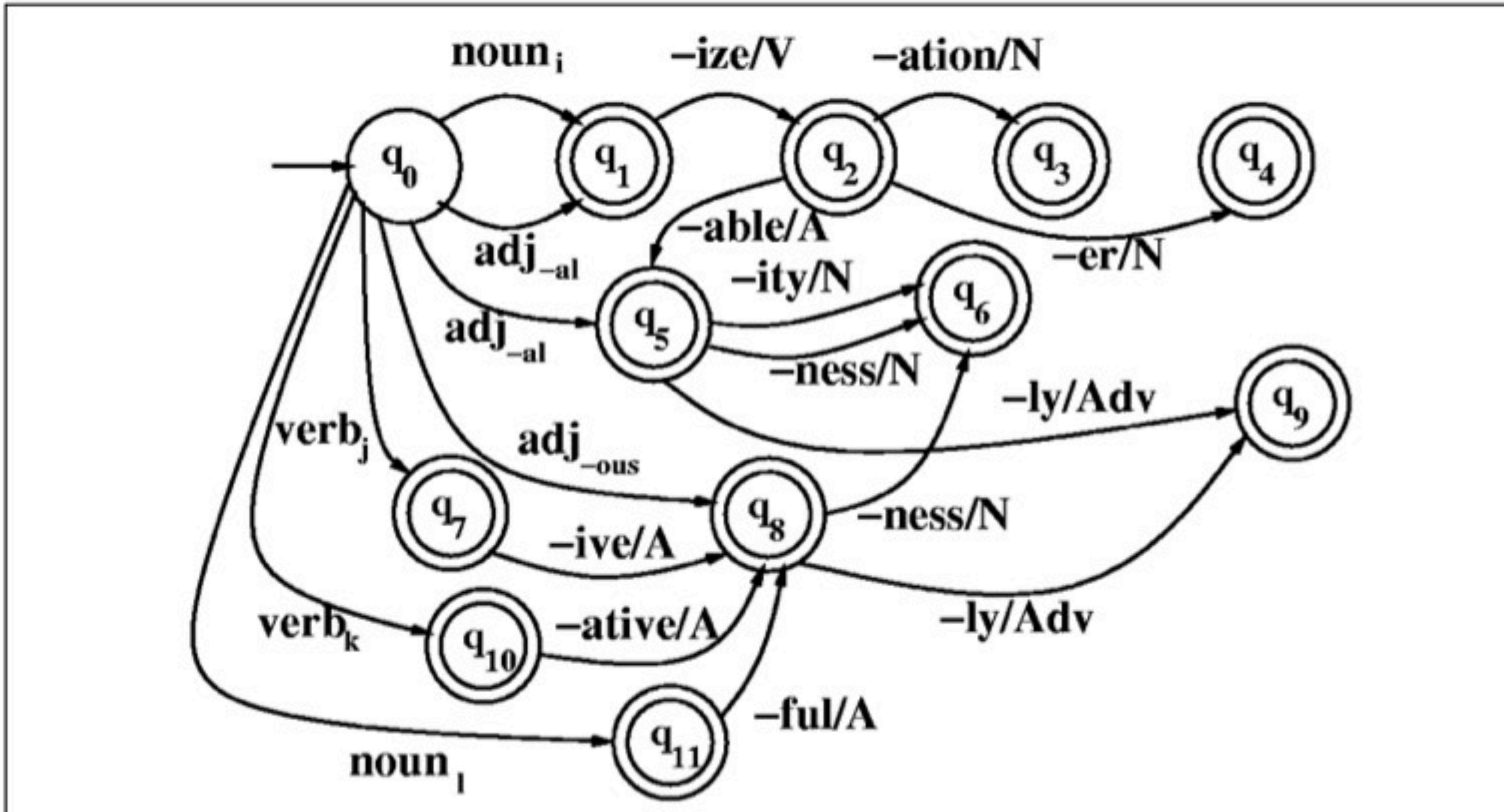
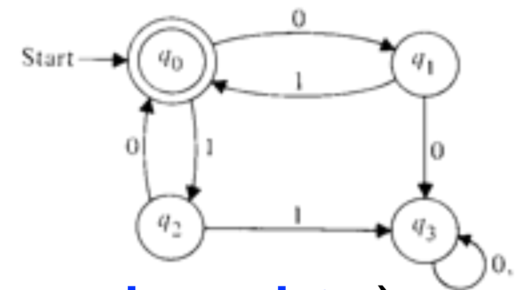


Figure 3.6 An FSA for another fragment of English derivational morphology.

Membership and Complement

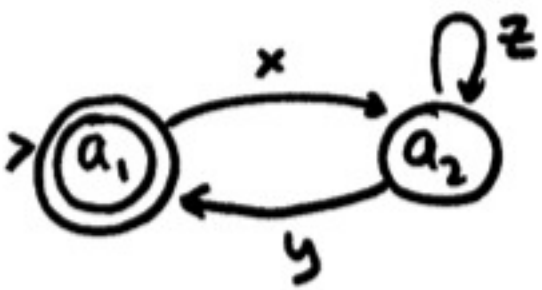
- deterministic FSA: iff no state has two exiting transitions with the same label. (DFA)

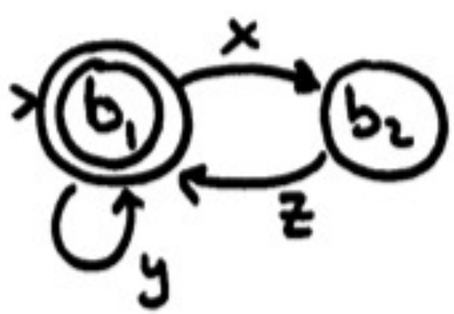


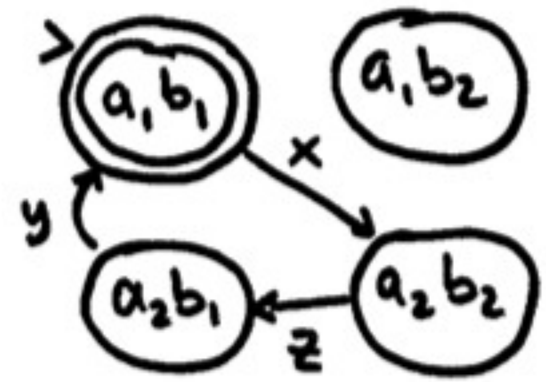
- the language L of a DFA D: $L = L(D)$
- how to check if a string w is in $L(D)$? (membership)
  - linear-time: follow transitions, check finality at the end
  - no transition for a char means "into a trap state"
- how to construct complement DFA? $L(D') = \neg L(D)$
  - super easy: just reverse the finality of states :)
  - note that "trap states" also become final states

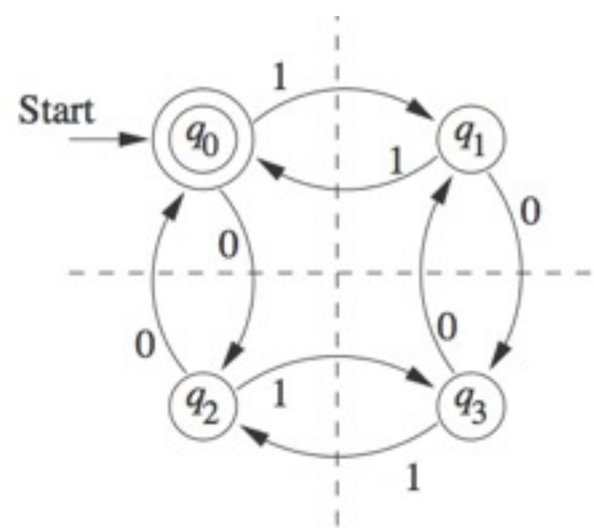
Intersection

- construct D s.t. $L(D) = L(D_1) \cap L(D_2)$
- state-pair ("cross-product") construction
  - intersected DFA: $|Q| = |Q_1| \times |Q_2|$





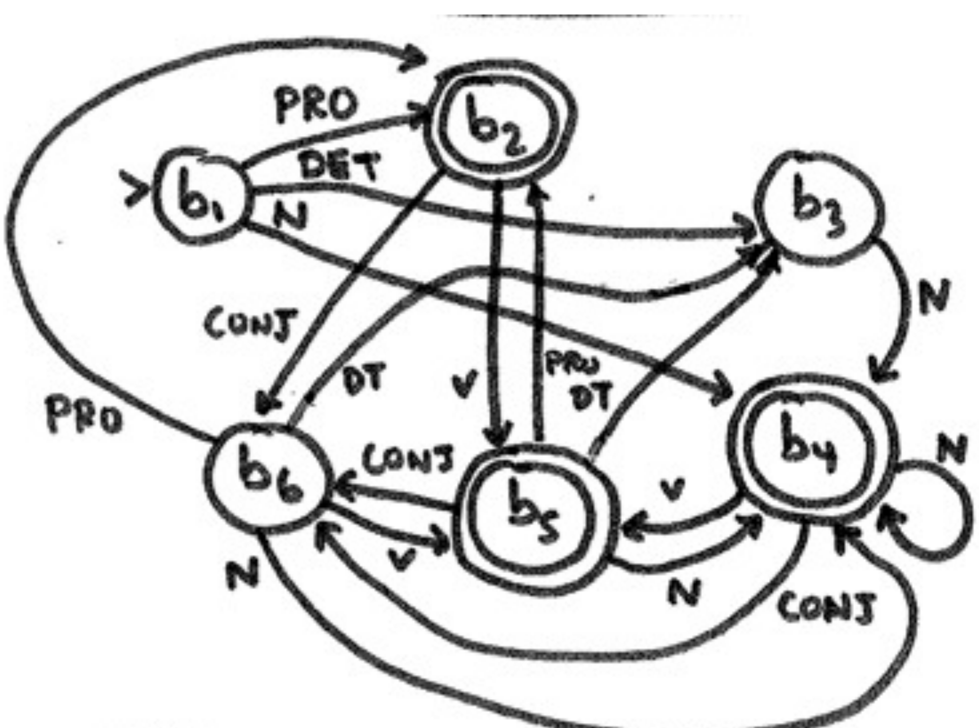




Linguistic Example

- DFA *A*: all interpretations of "he hopes that this works"

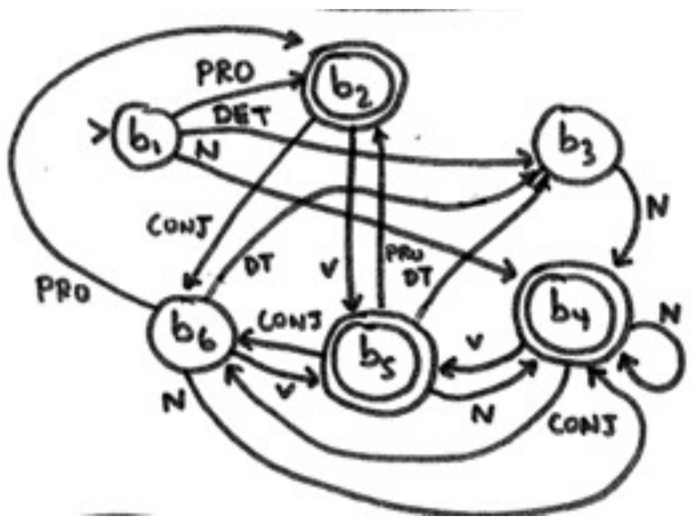
- DFA *B*: all legal English category sequences (simplified)

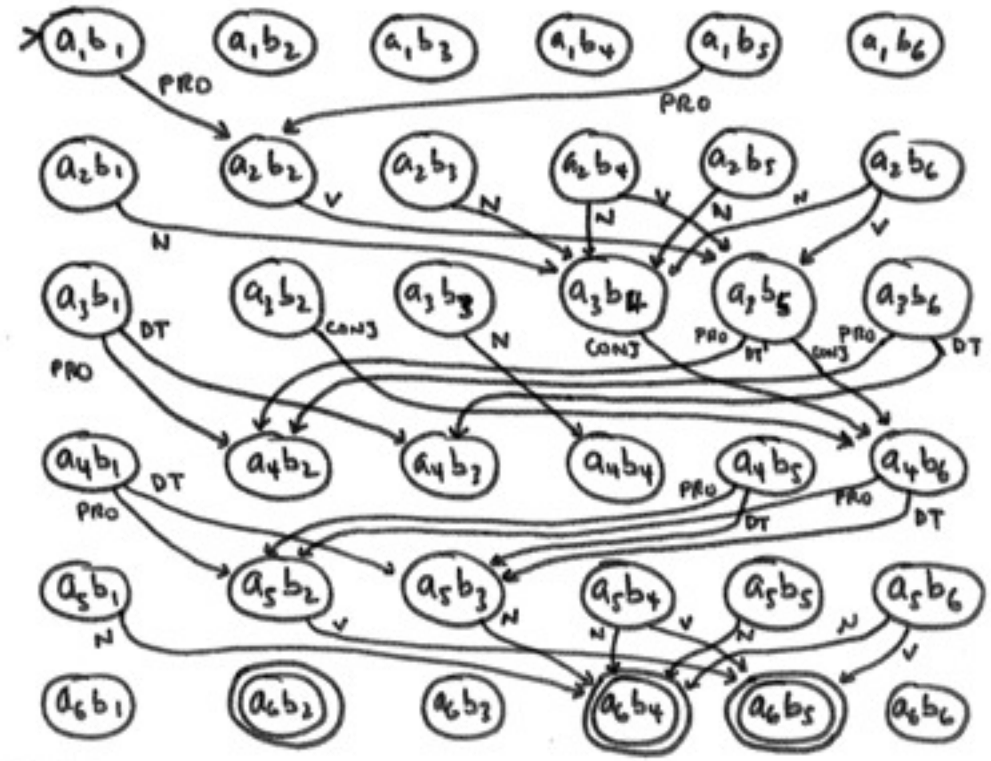


what do these states mean?

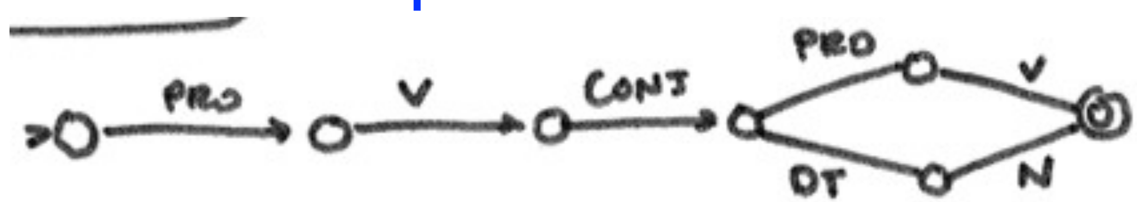
what will $A \cap B$ mean?

- intersection by state-pair (“product”) construction





- cleanup: he hopes that this works



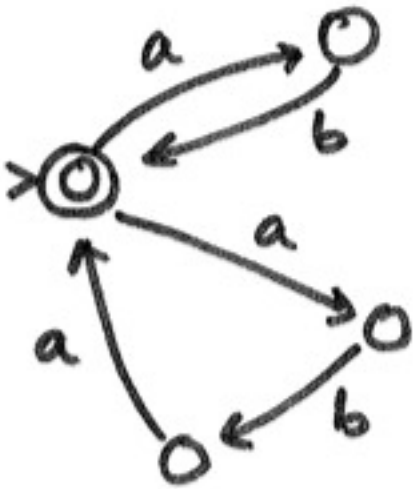
- this is part-of-speech tagging! (with a bigram model)

Union

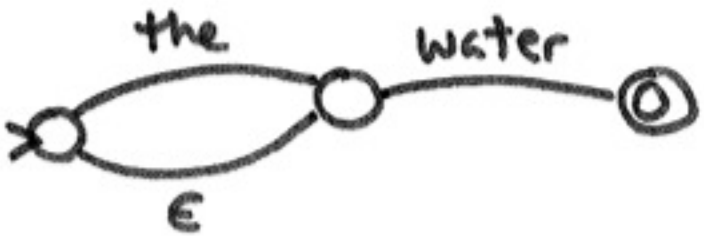
- easy, via De Morgan's Law: $L_1 \cup L_2 = \neg (\neg L_1 \cap \neg L_2)$
- or, directly, from the product construction again
- what are the final states?
  - could end in either language: $Q_2 \times F_1 \cup Q_1 \times F_2$
  - same De Morgan: $\neg ((Q_1 \backslash F_1) \cap (Q_2 \backslash F_2)) = \neg (\neg F_1 \cap \neg F_2)$

Non-Deterministic FSAs

- L = { *all strings of repeated instances of* ab *or* aba }
  - hard to do with a deterministic FSA!
  - e.g., abababaababa



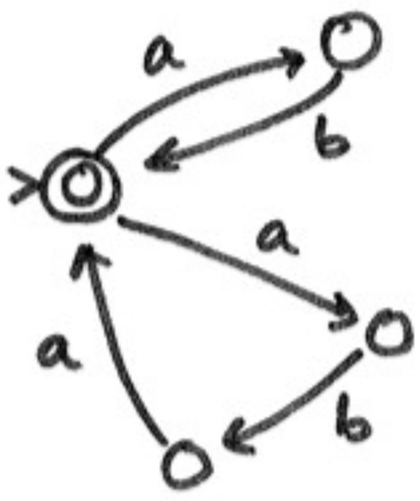
- epsilon transition (no symbol)



- there is algorithm to determinize a DFA
  - blow up the state-space exponentially

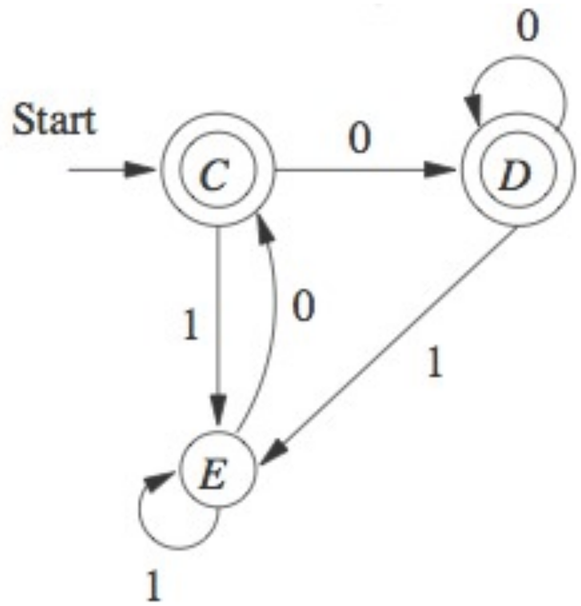
Determinization Example

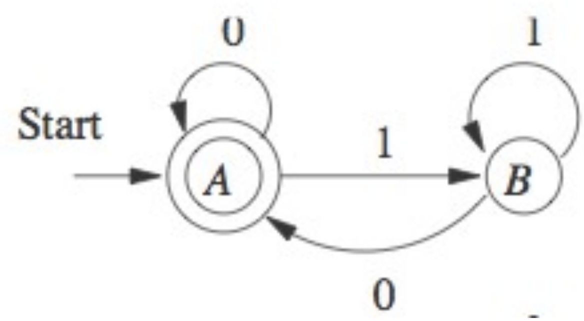
- determinization by subset construction (2^n)



Minimization and Equivalence

- each DFA (and NFA) can be reduced to an equivalent DFA with minimal number of states
  - based on "state-pair equivalence test"
  - can be used to test the equivalence of DFAs/NFAs



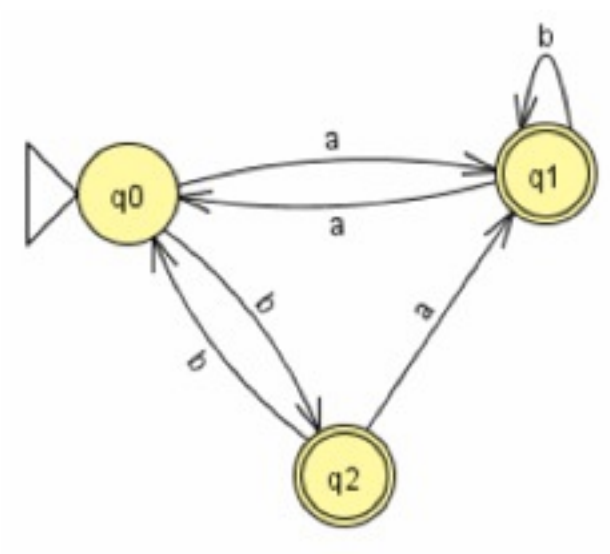


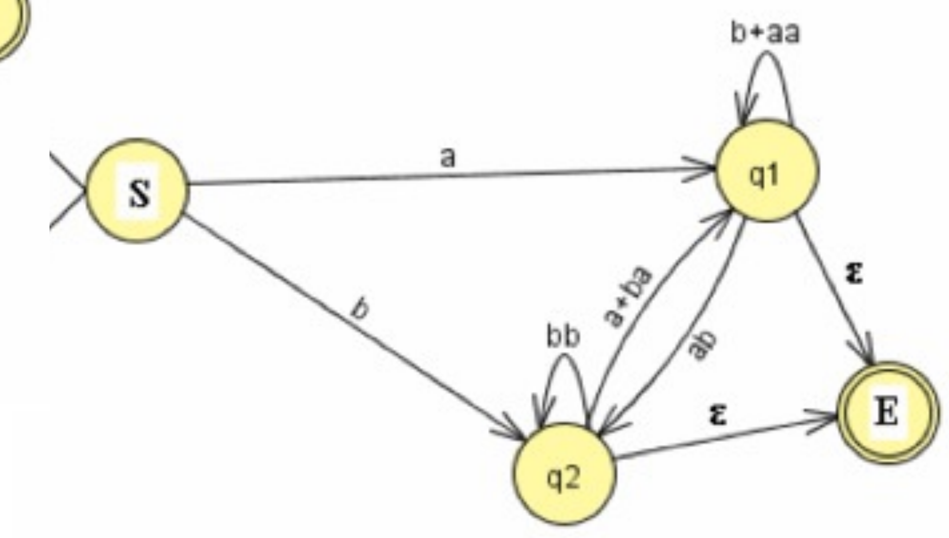
Advantages of Non-Determinism

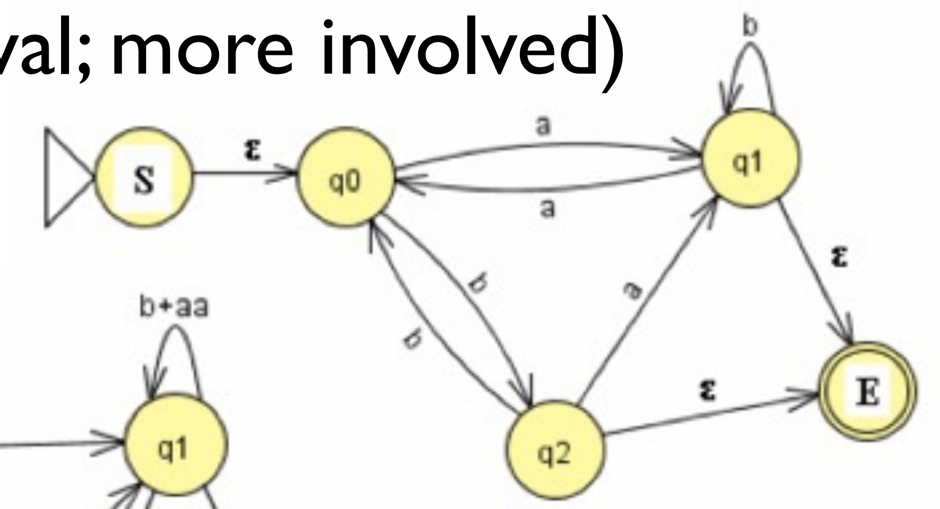
- union (and intersection also?)
- concatenation: $L_1L_2 = \{ xy \mid x \text{ in } L_1, y \text{ in } L_2\}$
- membership problem
  - much harder: exp. time => rather determinize first
- complement problem (similarly harder)
- but is NFA more expressive than DFA?
  - NO, because you can always determinize an NFA
- NFA: more "intuitive" representation of a language
- mDFA: "compact (but less intuitive) encoding"

FSAs vs. Regular Expressions

- RE operators: R^*, R_1+R_2, R_1R_2
- RE => NFA (by recursive translation; easy)
- NFA => RE (by state removal; more involved)







- RE <=> NFA <=> DFA <=> mDFA

Wrap-up

- machineries: (infinite) languages, DFAs, NFAs, REs
  - why and when non-determinism is useful
- constructions/algorithms
  - state-pair construction: intersection and union
    - quadratic time/space
  - subset construction: determinization
    - exponential time/space
  - briefly mentioned: minimization and RE <=> NFA
    - see Hopcroft et al textbook for details

Quick Review

- how to detect if a DFA accepts any string at all?
  - how about empty string?
  - how about all strings?
- how about an NFA?
- how to design a *reversal* of a DFA/NFA?

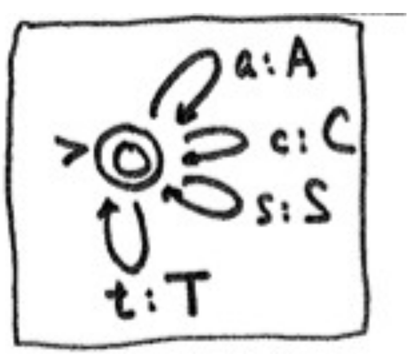
Finite-State Transducers

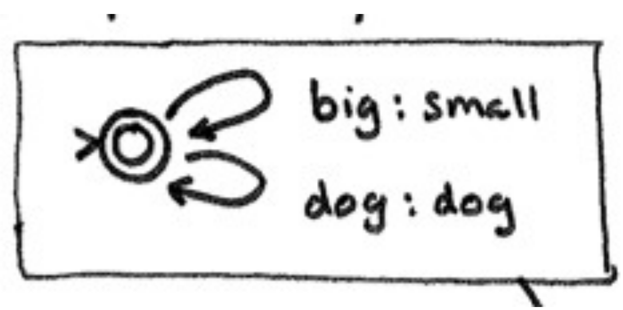
- FSAs are "acceptors" (set of strings as a language)
- FSTs are "converters"
  - compactly encoding set of string pairs as a relation
- capitalizer: { <cat, CAT>, <dog, DOG>, ...}
- pluralizer: {<cat, cats>, <fly, flies>, <hero, heroes>...}

Formal Definition

- a finite-state transducer *T* is a tuple (*Q*, Σ, Γ, *I*, *F*, δ) such that:

▪ *Q* is a finite set, the set of *states*;
▪ Σ is a finite set, called the *input alphabet*;
▪ Γ is a finite set, called the *output alphabet*;
▪ *I* is a subset of *Q*, the set of *initial states*;
▪ *F* is a subset of *Q*, the set of *final states*; and
▪ $\delta \subseteq Q \times (\Sigma \cup \{\epsilon\}) \times (\Gamma \cup \{\epsilon\}) \times Q$ is the *transition relation*.



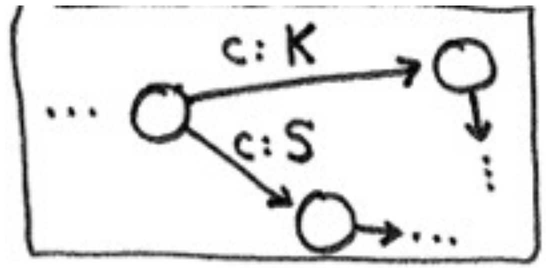


Examples

- text-to-sound: {<c a t, K AE T>, <d o g, D AW G>,
  <b e a r, B EH R>, <b a r e, B EH R>...}
  - (easy for Spanish/Italian, medium for French, hard for English!)
- POS tagger: {<I saw the cat, PRO V DT N>, ...}
- transliterator: { <b u s h, 布 什>, <o b a m a, 奥 巴 马>, ...}
  bu shi *ao ba ma*
- translator: { <he is in the house, el está en la casa>,
  <he is in the house, está en la casa>, ... }
- notice the many-to-many relation (not a function)
- but is this real translation? NO, there are no reorderings!
  - FSMs are best for morphology; we need CFGs for syntax

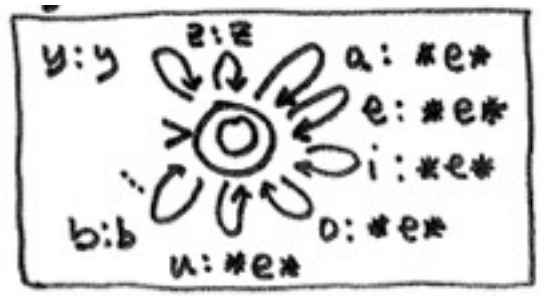
Non-Determinism in FSTs

- ambiguity



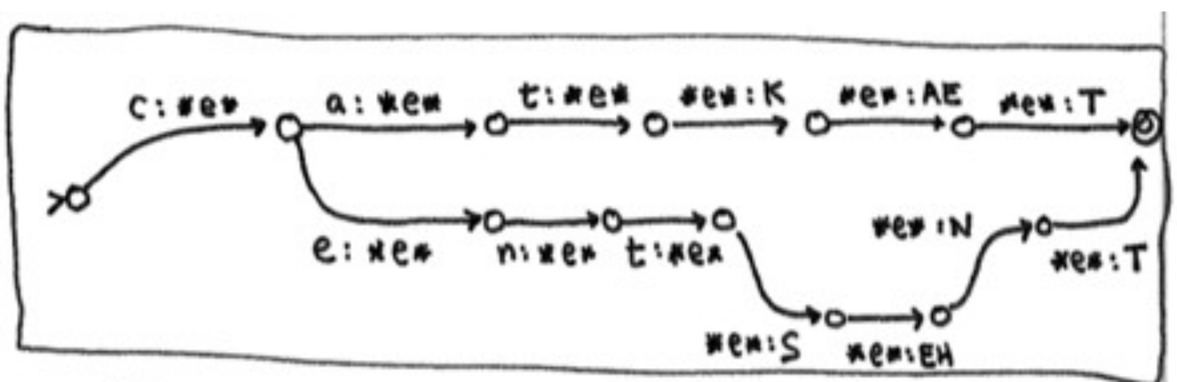
charactr "c"
pronounced as either
K sound or S sound

- optionality



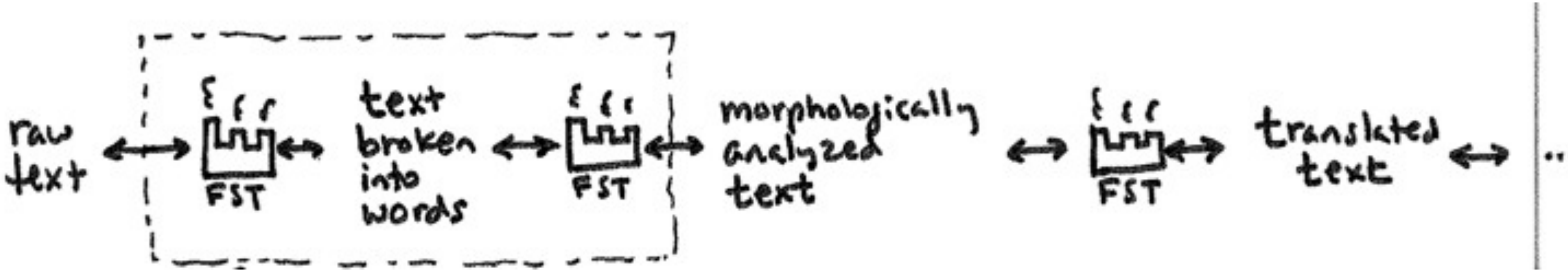
  - important because in/out often have different lengths

- delayed decision via epsilon transition



"input word, output pronunciation"

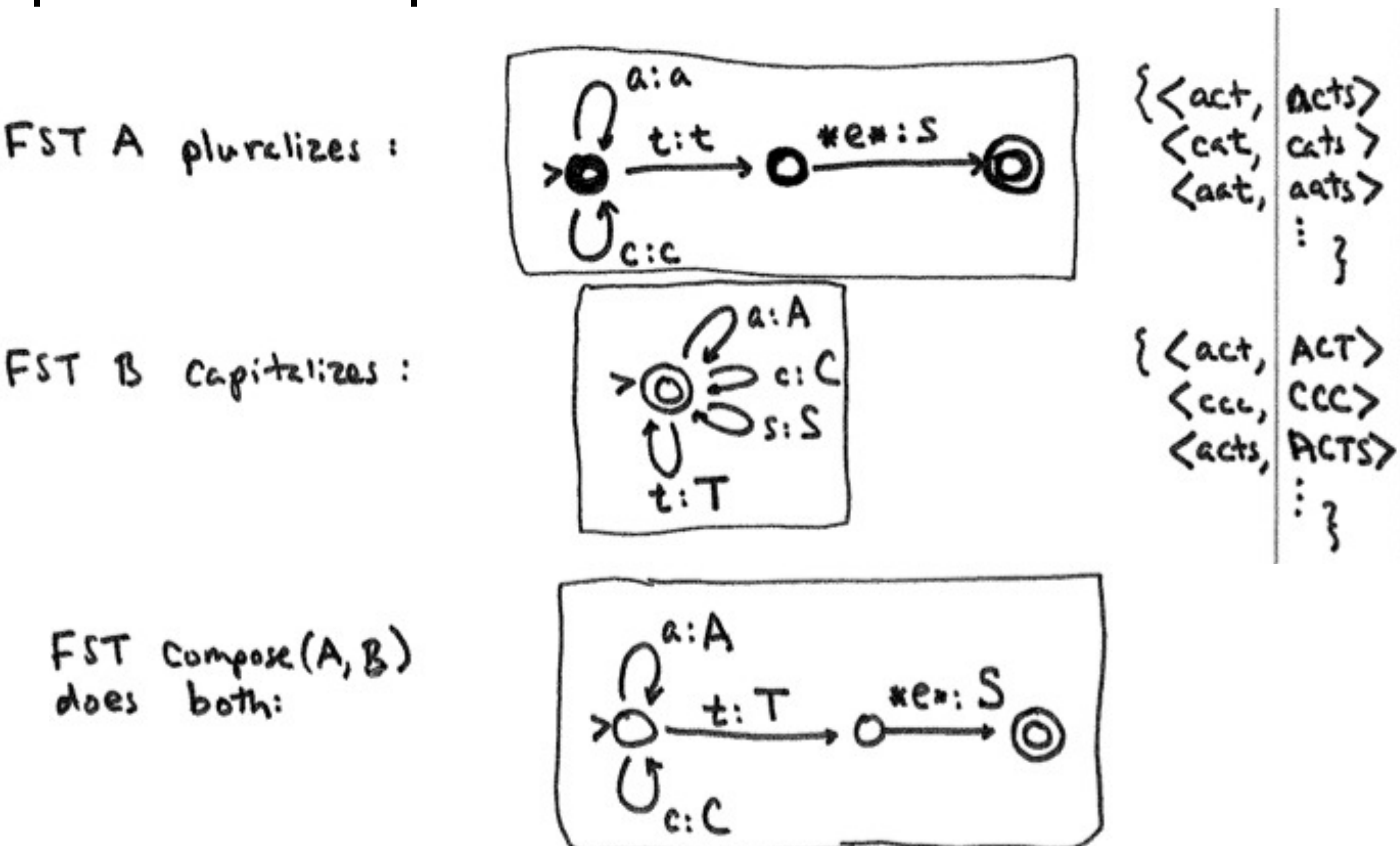
Central Operation: Composition



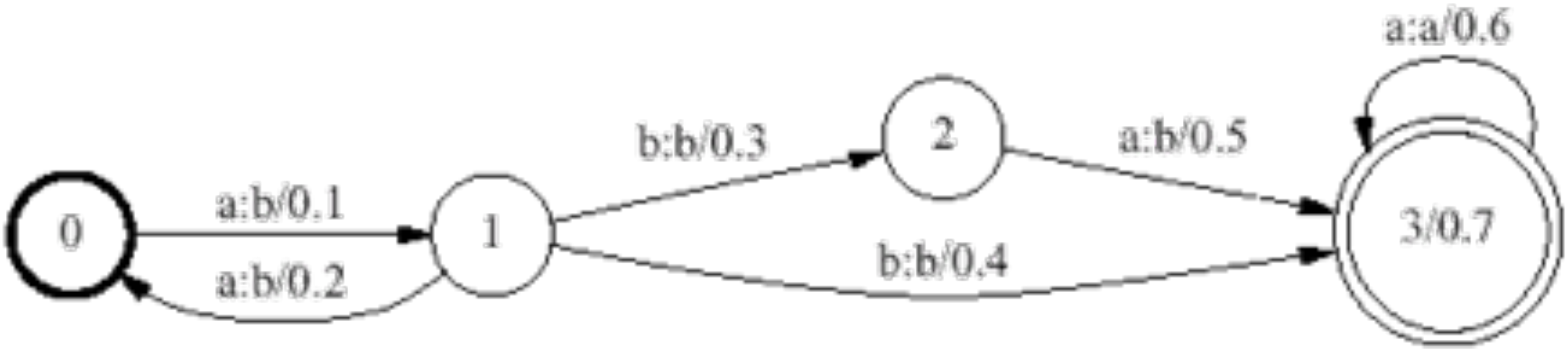
- language processing is often in cascades
  - often easier to tackle small problems separately
- each step: T(A) is the relation (*set of string pairs*) by A
  - <x, y> in T(A) means $x \sim_A y$
- compose (A, B) = C
  - <x, y> in T(C) iff. ∃ z: <x, z> in T(A) and <z, y> in T(B)

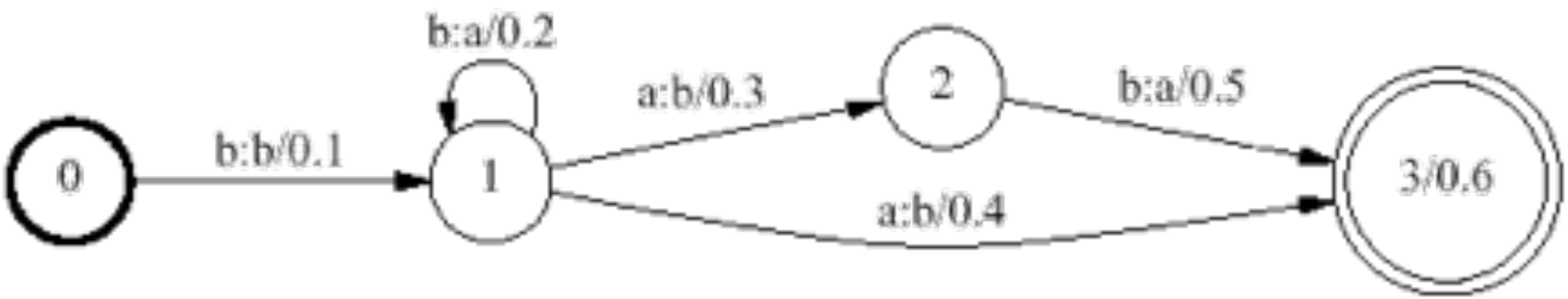
Simple Example

- pluralizer + capitalizer

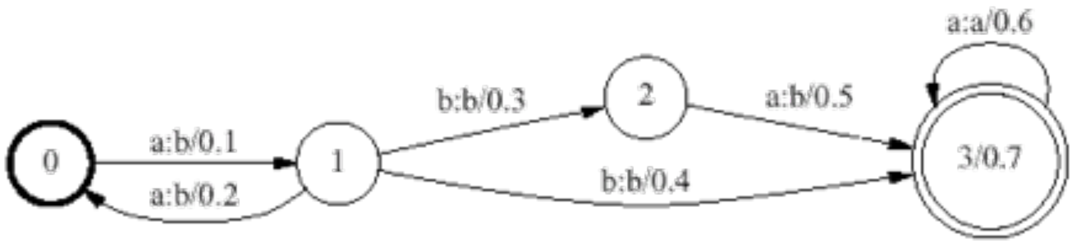


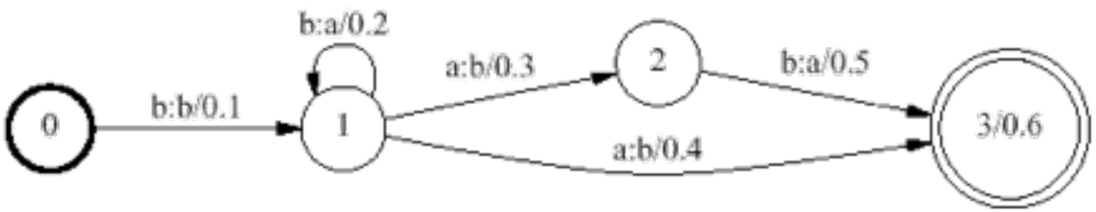
How to do composition?

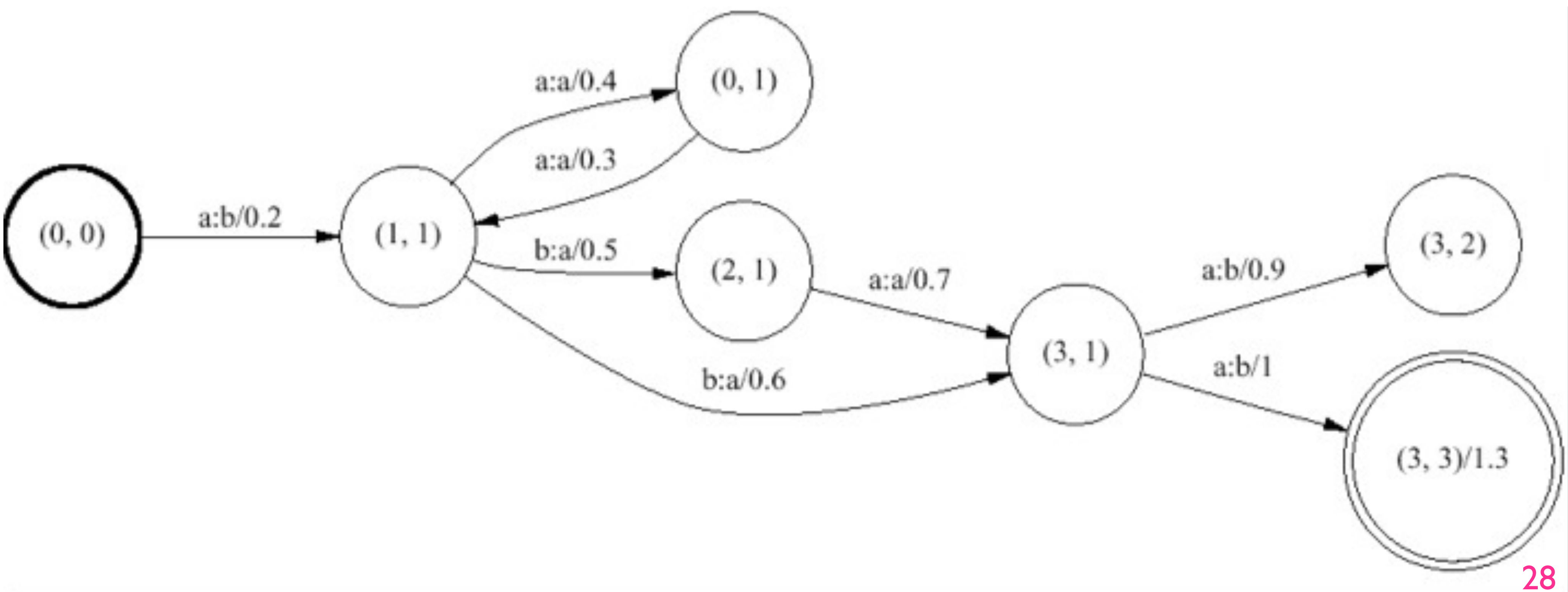




How to do composition?





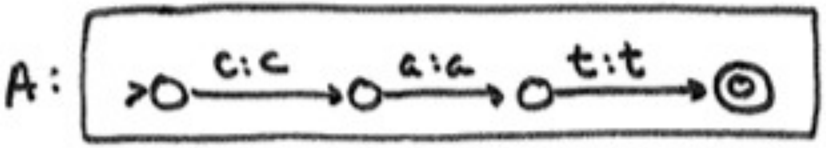


composition is like intersection?

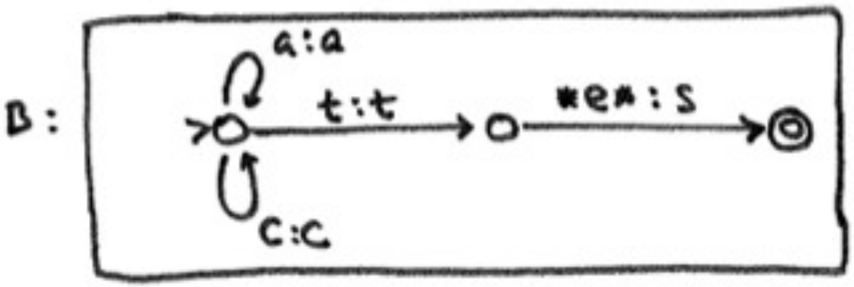
- both use cross-product ("state-pair") construction
- indeed: intersection is a special case of composition
  - FSA is a special FST with identity output! (a => a:a)
  - $A \cap B = \mathrm{proj}_{in}(\ \mathrm{Id}(A) \circ \mathrm{Id}(B)\ )$
- what about FSAs composed with FSTs?
  - FSA ◦ FST --- get output(s) from certain input(s)
    - <x, z>: ∃ y s.t. <x, y> in T(Id(A)) and <y,z> in T(B)
    - but y=x => <x, z>: x in L(A) and <x,z> in T(B)
  - FST ◦ FSA --- get input(s) for certain output(s)

Get Output

e.g., pluralize "cat"

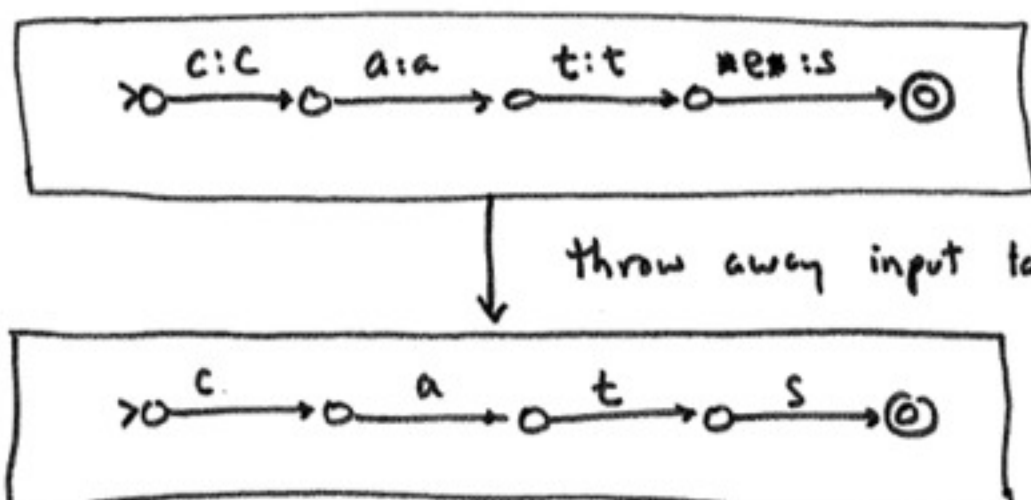


$\{\langle cat, cat\rangle\}$



$\{\langle cat, cats\rangle, \langle act, acts\rangle, \ldots\}$

Compose (A, B) includes $\langle x, y\rangle$ if $\exists z: \langle x, z\rangle \in A$ & $\langle z, y\rangle \in B$

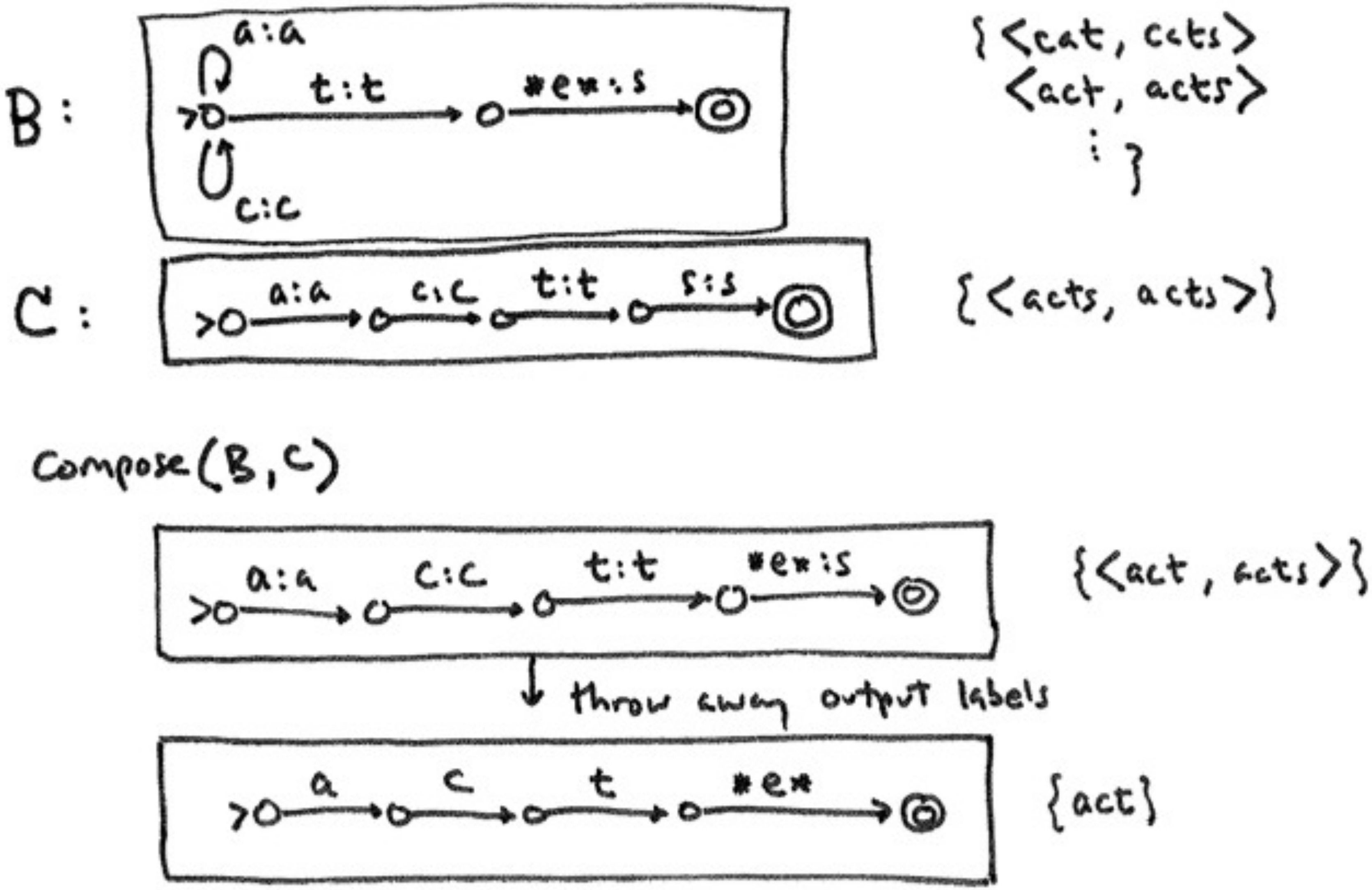


FST $\{\langle cat, cats\rangle\}$

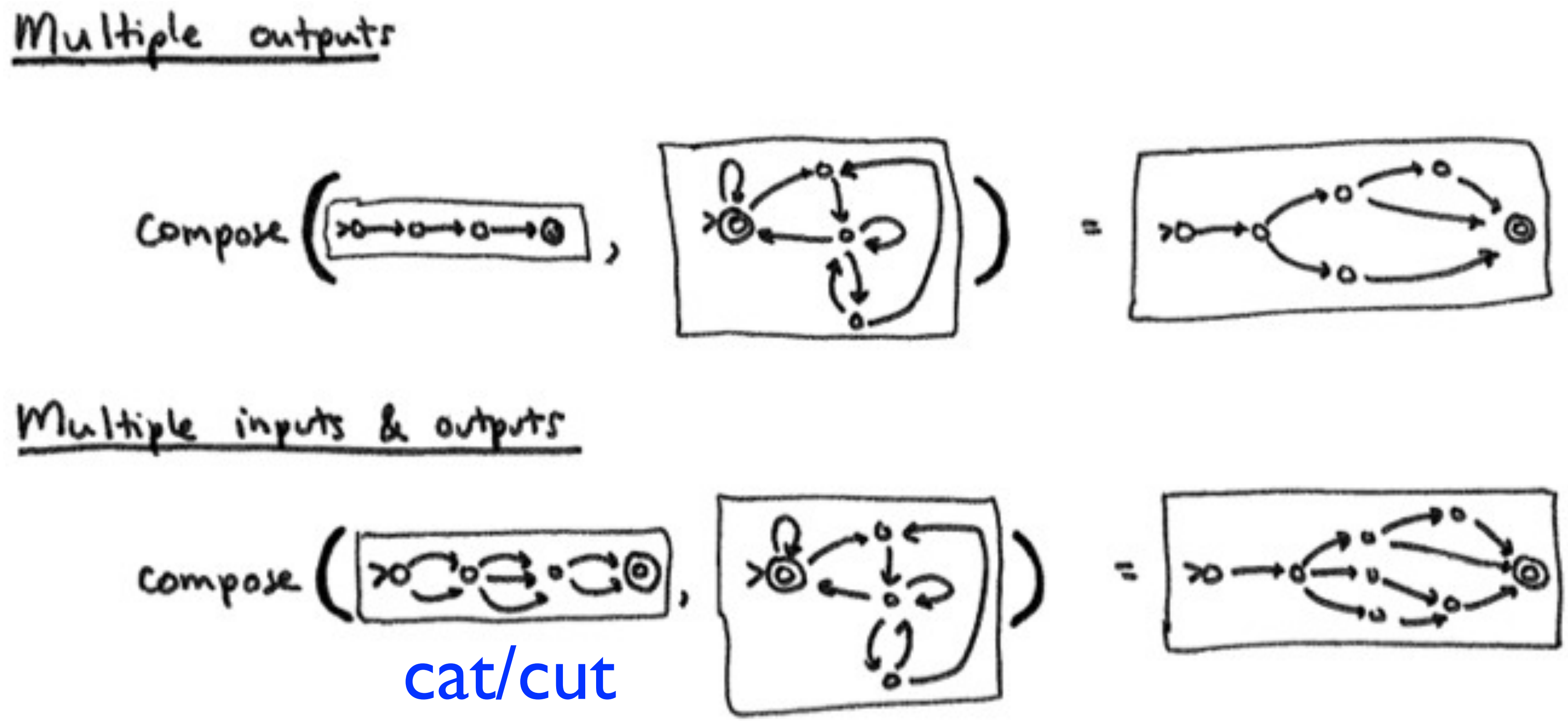
FSA $\{cats\}$

Get Input

- morphological analysis (e.g. what is “acts” made from)



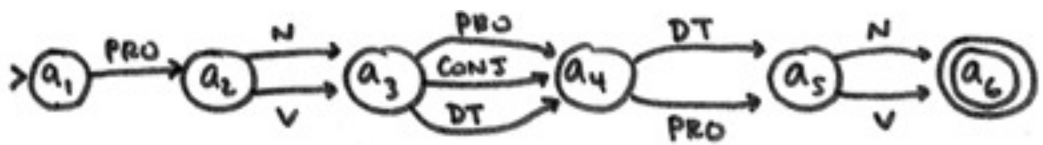
Multiple Outputs

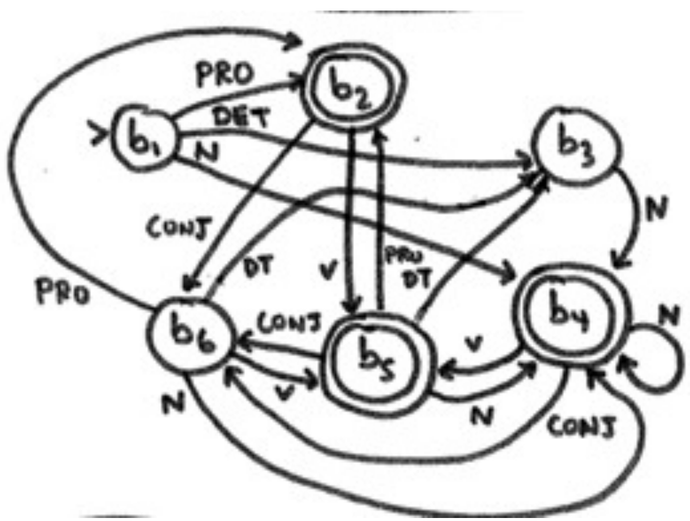


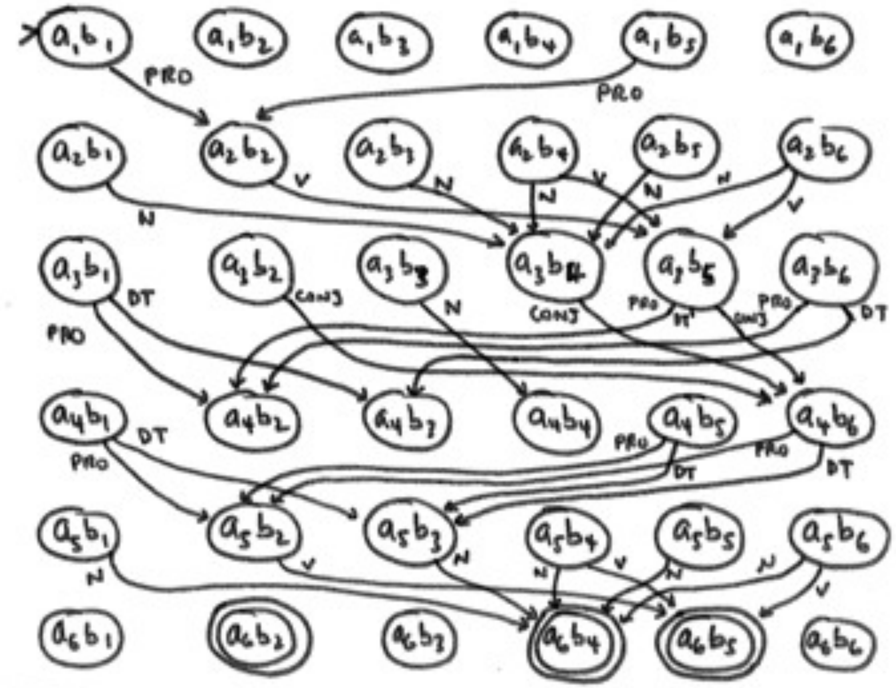
- text-to-sound: {<c a t, K AE T>, <d o g, D AW G>,
  <b e a r, B EH R>, <b a r e, B EH R>...}
- translator: { <he is in the house, el está en la casa>,
  <he is in the house, está en la casa>, ... }

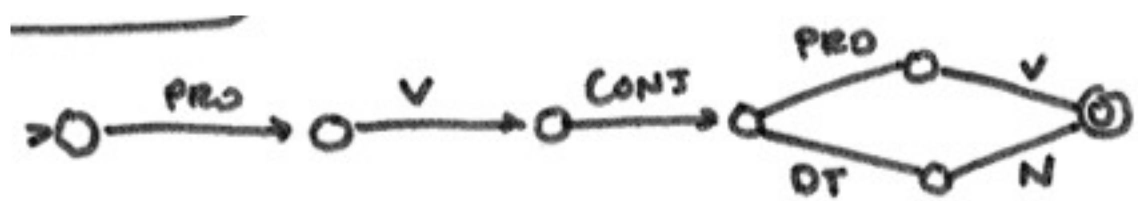
POS Tagging Revisited

- he hopes that this works



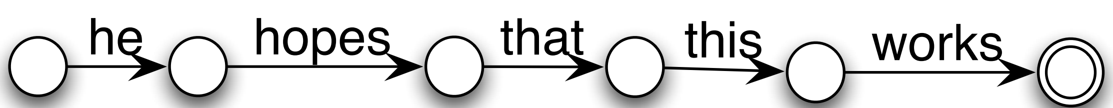




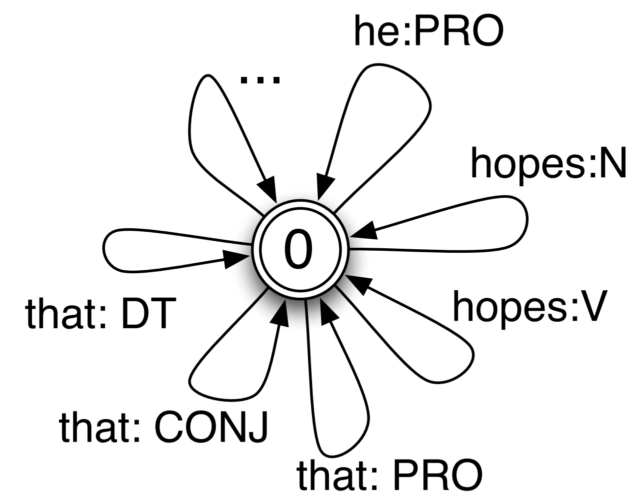


Redo POS Tagging via composition

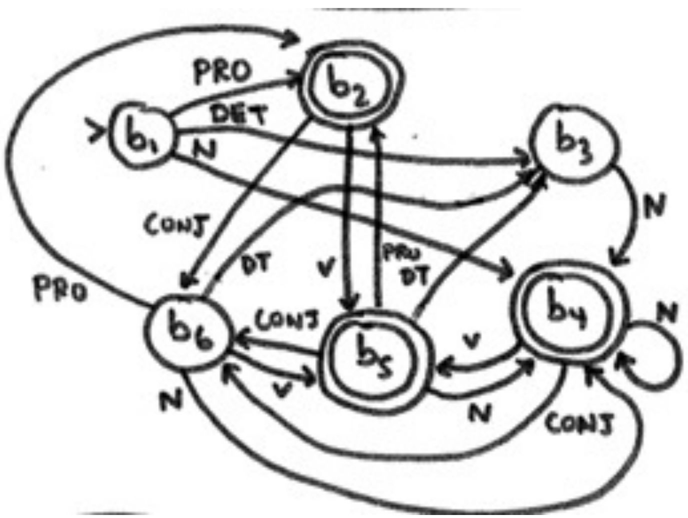
FST A: sentence



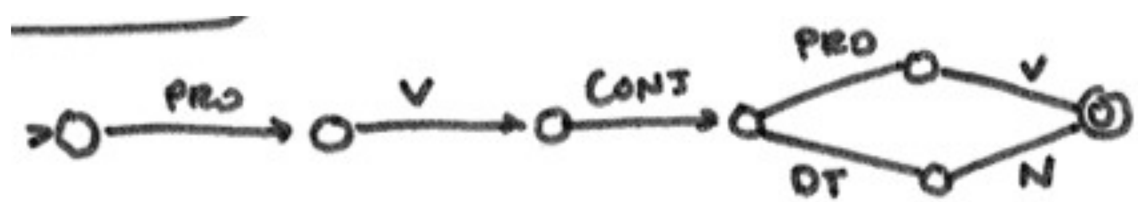
FST B: lexicon



FST C: POS bigram LM



$$\mathrm{proj}_{out}(A \circ B \circ C) =$$



Q: how about $A \circ (B \circ C)$? what is $B \circ C$?