Language Technology

CUNY Graduate Center Spring 2013

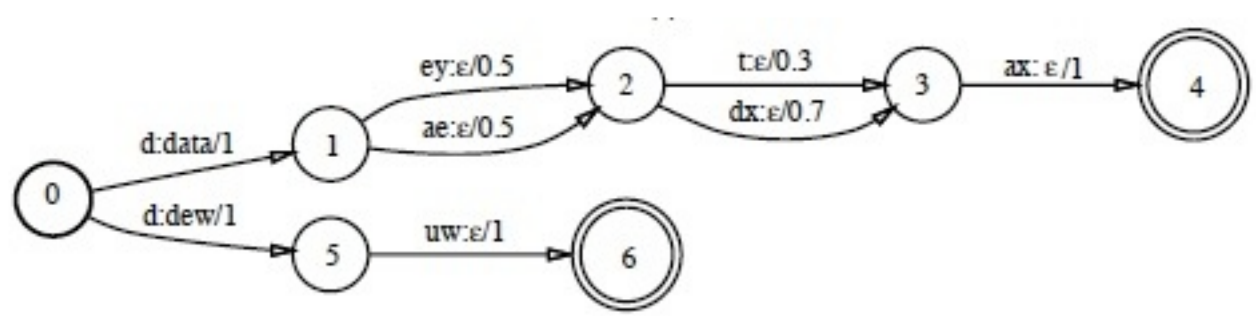
Unit 1: Sequence Models

Lectures 5-6: Language Models and Smoothing

required

hard

optional



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Python Review: Styles

• do not write ...	when you can write ...
`for key in d.keys():`	`for key in d:`
`if d.has_key(key):`	`if key in d:`

`i = 0`
`for x in a:`
`    ...`
`    i += 1`
	`for i, x in enumerate(a):`

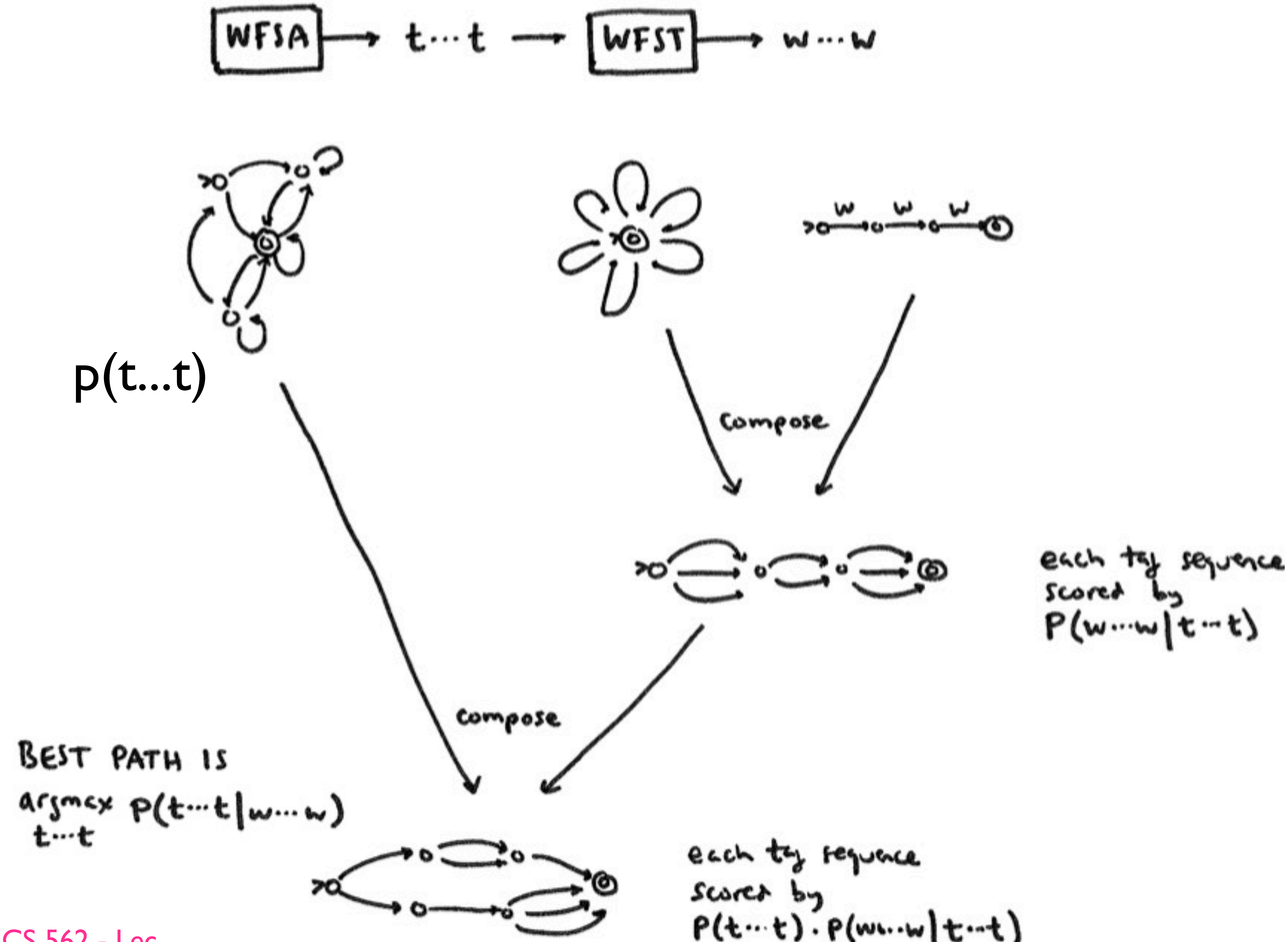
`a[0:len(a) - i]`	`a[:-i]`
`for line in \` `    sys.stdin.readlines():`	`for line in sys.stdin:`

`for x in a:`
`    print x,`
`print`
	`print " ".join(map(str, a))`

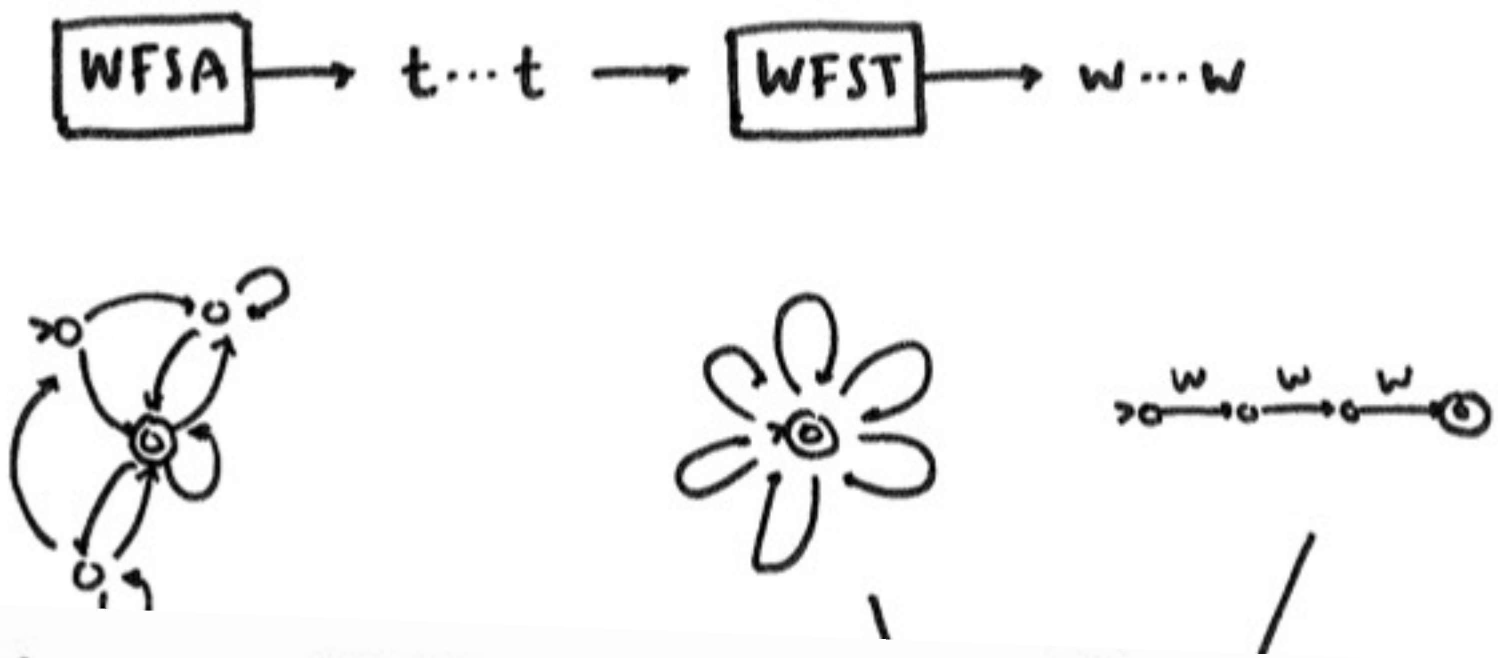
`s = ""`
`for i in range(lev):`
`    s += " "`
`print s`
	`print " " * lev`

Noisy-Channel Model

Noisy-Channel Model

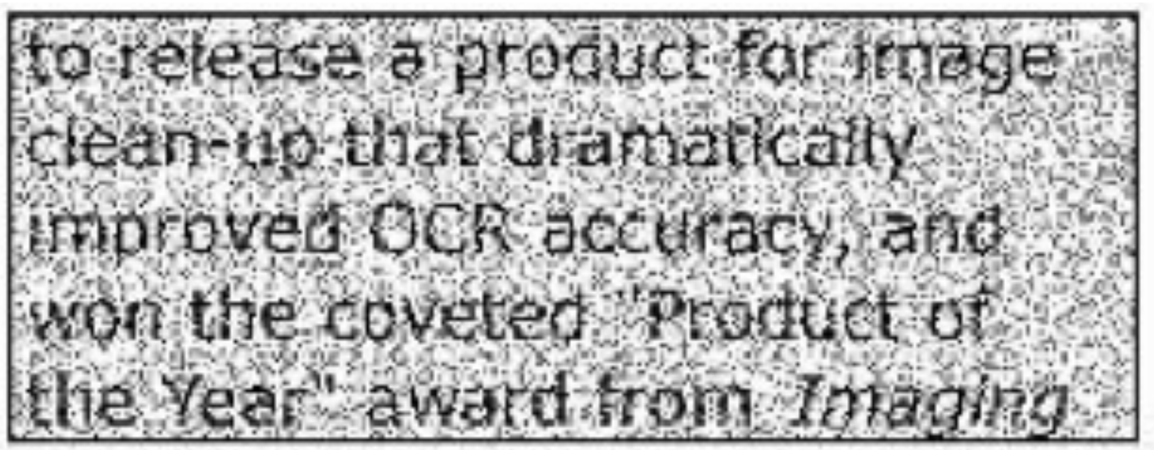


Applications of Noisy-Channel



Application	Input	Output	p(i)	p(o|i)
Machine Translation	L_1 word sequences	L_2 word sequences	p(L_1) in a language model	translation model
Optical Character Recognition (OCR)	actual text	text with mistakes	prob of language text	model of OCR errors
Part Of Speech (POS) tagging	POS tag sequences	English words	prob of POS sequences	p(w|t)
Speech recognition	word sequences	speech signal	prob of word sequences	acoustic model
spelling correction	correct text	text with mistakes	prob. of language text	noisy spelling

Noisy Channel Examples


to release a product for image
clean-up that dramatically
improved OCR accuracy, and
won the coveted "Product of
the Year" award from Imaging

Th qck brwn fx jmps vr th lzy dg.
Ths sntnc hs ll twnty sx lttrs n th lphbt.

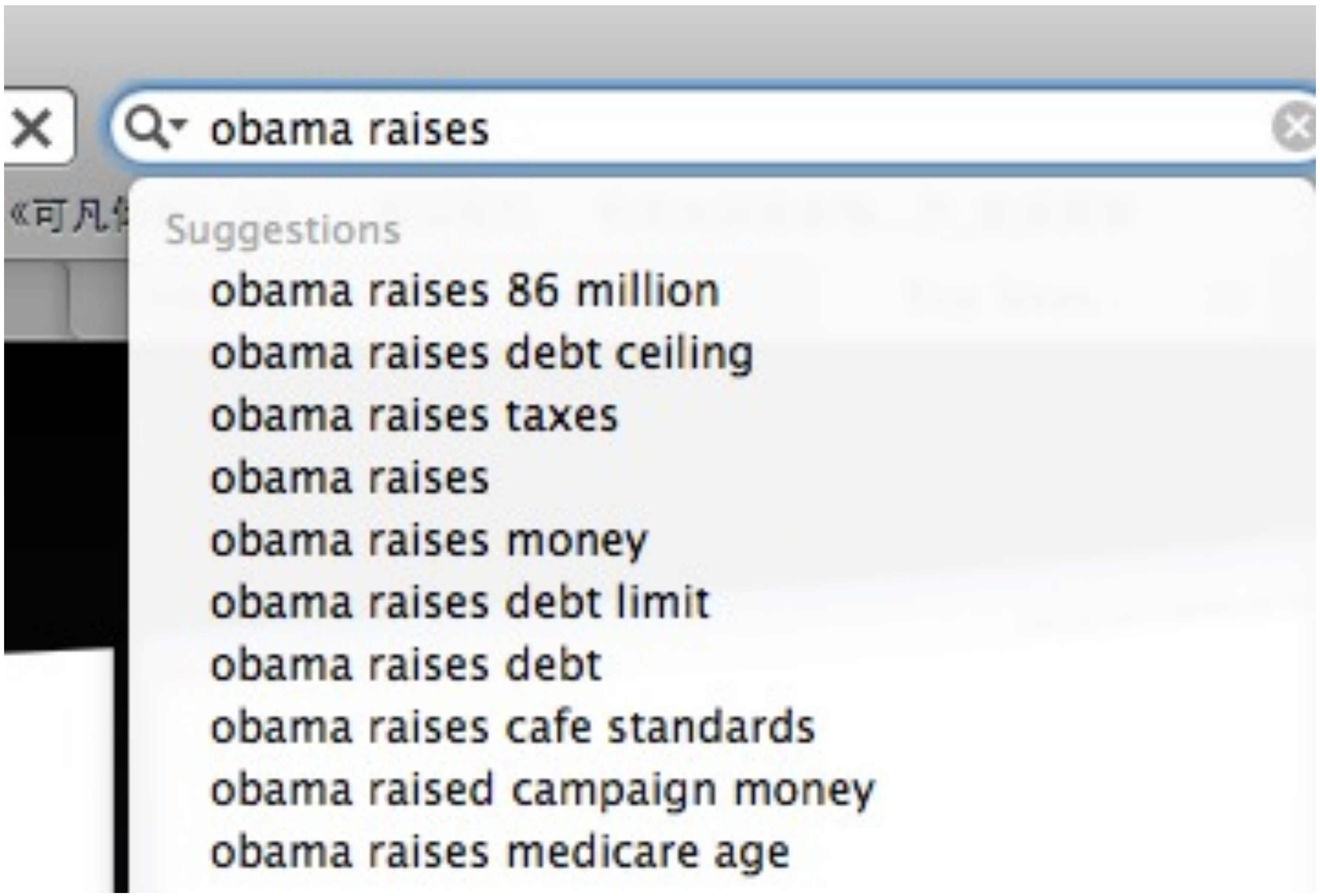
I cnduo't bvleiee taht I culod aulaclty uesdtannrd waht I was rdnaieg. Unisg the icndeblire pweor of the hmuan mnid, aocdcrnig to rseecrah at Cmabrigde Uinervtisy, it dseno't mttaer in waht oderr the lterets in a wrod are, the olny irpoamtnt tihng is taht the frsit and lsat ltteer be in the rhgit pclae.

Therestcanbeatotalmessandyoucanstillreaditwithoutaproblem.Thisisbecausethehumanminddoesnotreadeveryletterbyitself,butthewordasawhole.

Noisy Channel Examples

Language Model for Generation

- search suggestions



Language Models

- problem: what is $P(\mathbf{w}) = P(w_1\ w_2 \ldots w_n)$?
  - ranking: P(an apple) > P(a apple)=0, P(he often swim)=0
  - prediction: what's the next word? use $P(w_n \mid w_1 \ldots w_{n-1})$
    - Obama gave a speech about _____ .

sequence prob, not just joint prob.

- $P(w_1\ w_2 \ldots w_n) = P(w_1)\ P(w_2 \mid w_1) \ldots P(w_n \mid w_1 \ldots w_{n-1})$
- $\approx P(w_1)\ P(w_2 \mid w_1)\ P(w_3 \mid w_1\ w_2) \ldots P(w_n \mid w_{n-2}\ w_{n-1})$ trigram
- $\approx P(w_1)\ P(w_2 \mid w_1)\ P(w_3 \mid w_2) \ldots P(w_n \mid w_{n-1})$ bigram
- $\approx P(w_1)\ P(w_2)\ P(w_3) \ldots P(w_n)$ unigram
- $\approx P(w)\ P(w)\ P(w) \ldots P(w)$ 0-gram

Estimating *n*-gram Models

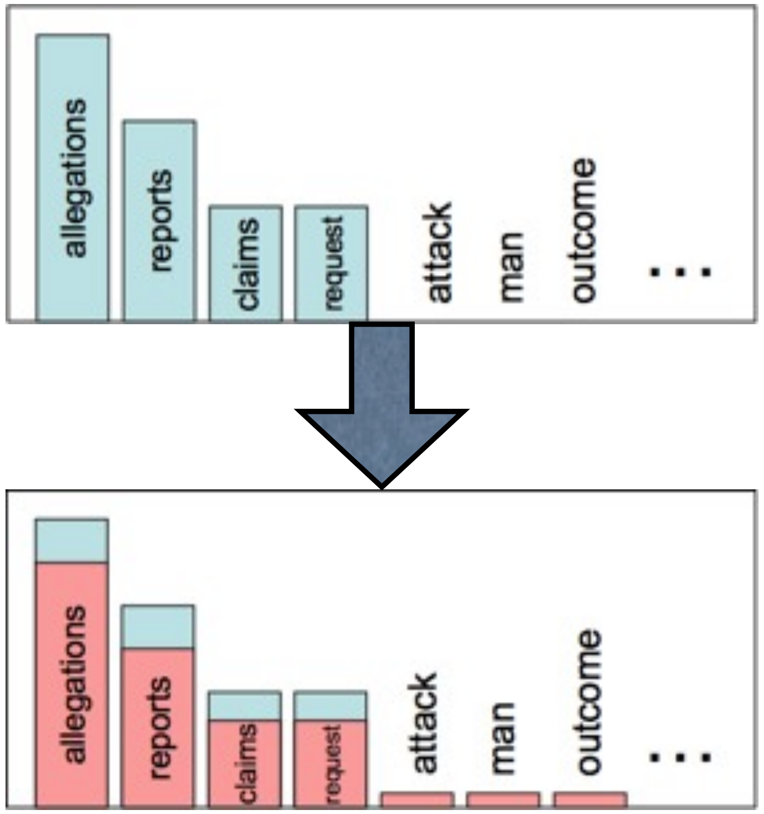
	“In person	she	was	inferior ~~superior~~	to	both	sisters”		
0-gram		10^{-6}	$\boxed{10^{-6}}$	10^{-6}	10^{-6}	10^{-6}	10^{-6}	≈	10^{-36}
unigram		.011	.015	.00005	.032	.0005	.0003	=	4×10^{-17}
bigram		.009	.122	0	.212	.0004	.006	=	
trigram		?	.5	0	?	0	0	=	
4-gram		?	?	0	?	?	?	=	

(textbook, table 6.3)

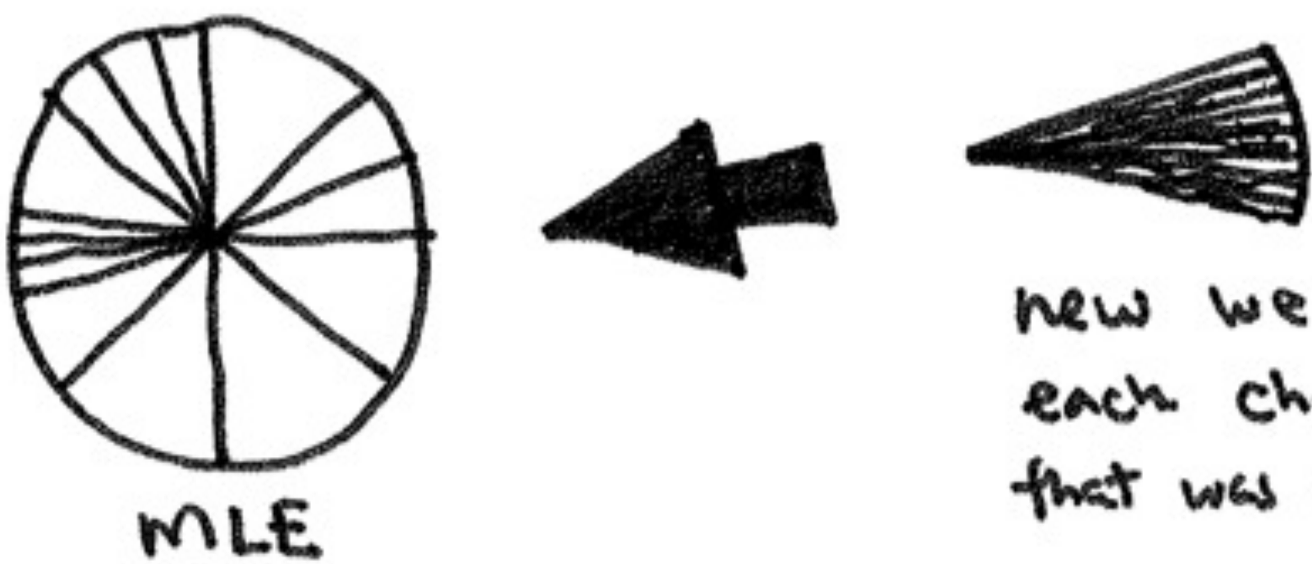
- maximum likelihood: $p_{ML}(x) = c(x)/N$; $p_{ML}(xy) = c(xy)/c(x)$
- problem: unknown words/sequences (unobserved events)
- sparse data problem
- solution: smoothing

Smoothing

- have to give some probability mass to unseen events
  - (by discounting from seen events)
- Q1: how to divide this wedge up?
- Q2: how to squeeze it into the pie?



(D. Klein)



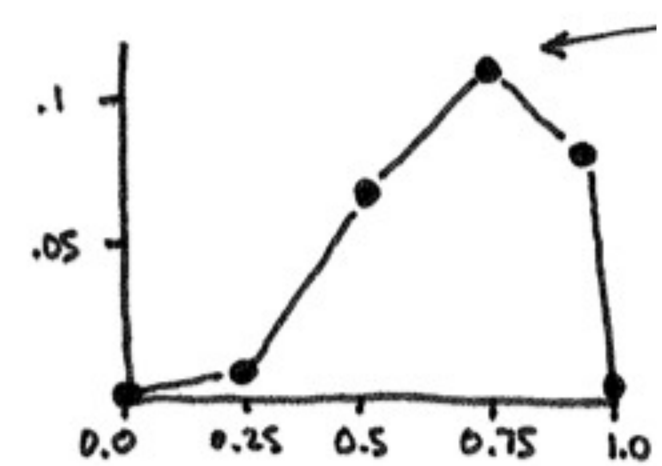
new wedge (one tiny slice for each character sequence of length < 20 that was never observed in training data)

ML, MAP, and smoothing

- simple question: what's P(H) if you see H H H H?
- always maximize posterior: what's the best m given d?
- with uniform prior, same as likelihood (explains data)
  - $\text{argmax}_m \; p(m|d) = \text{argmax}_m \; p(m) \, p(d|m)$ bayes, and p(d)=1
  - $= \text{argmax}_m \; p(d|m)$ when p(m) uniform

suppose d = H H T H

m_1	coin is unbiased	$P(d\|m) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 0.066$
m_2	coin is biased so that $P(H) = 3/4$	$P(d\|m) = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} = 0.105$
m_3	coin is biased so that $P(H) = 9/10$	$P(d\|m) = \frac{9}{10} \cdot \frac{9}{10} \cdot \frac{1}{10} \cdot \frac{9}{10} = 0.073$



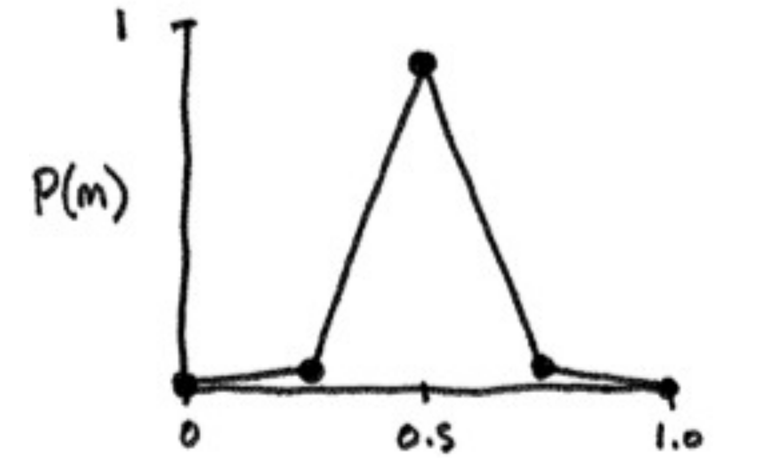
ML, MAP, and smoothing

m_1 coin is unbiased $P(m) = 0.90$

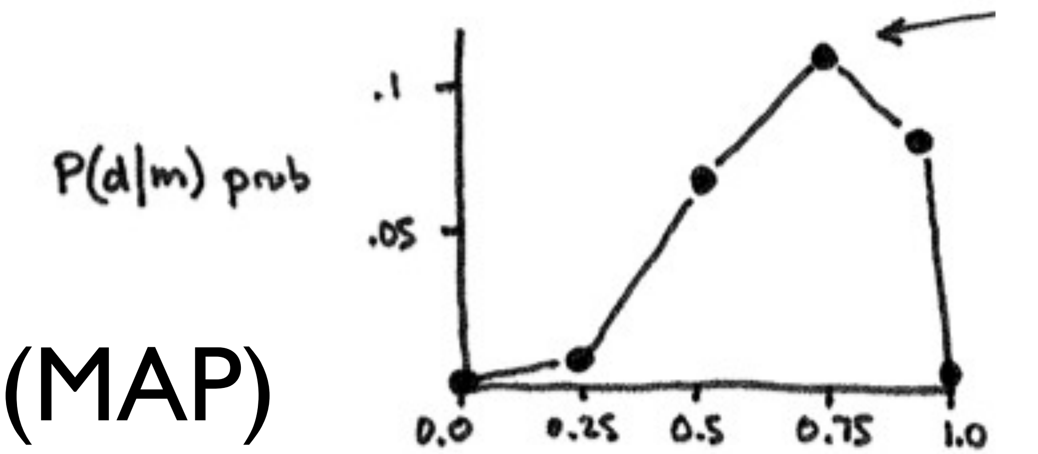
m_2 coin is biased 3/4 $P(m) = 0.01$

m_3 coin is biased 1/4 $P(m) = 0.01$

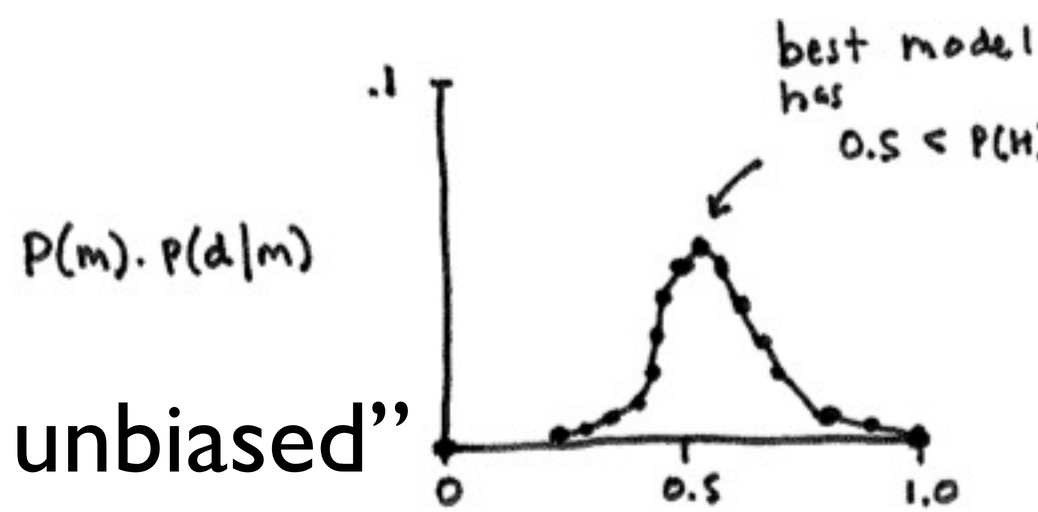
just "made up"!



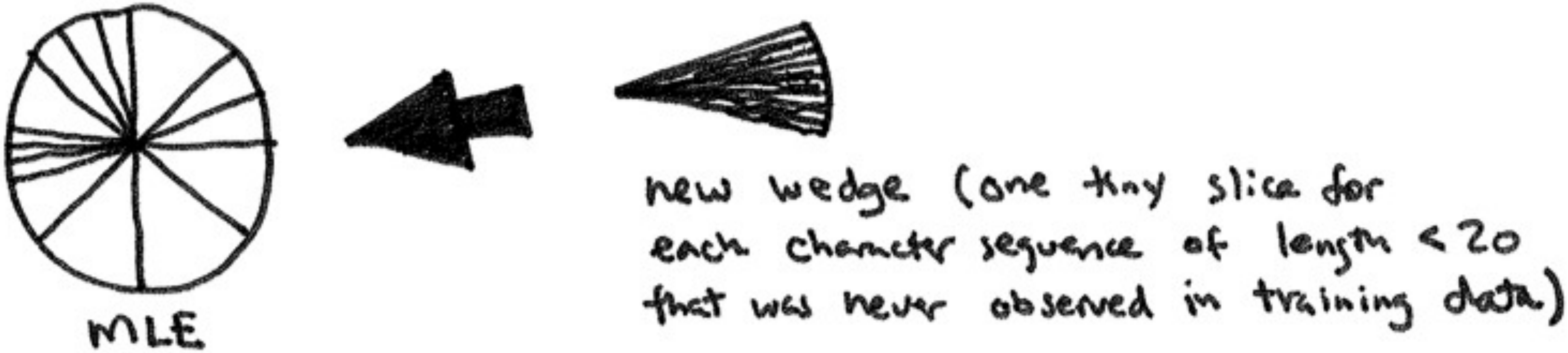
- what if we have arbitrary prior
  - like $p(\theta) = \theta\ (1-\theta)$



- maximum a posteriori estimation (MAP)
- MAP approaches MLE with infinite
- MAP = MLE + smoothing
  - this prior is just "extra two tosses, unbiased"
  - you can inject other priors, like "extra 4 tosses, 3 Hs"



Smoothing: Add One (Laplace)



- MAP: add a "pseudocount" of 1 to every word in Vocab
- $P_{lap}(x) = (c(x) + 1) / (N + V)$ V is Vocabulary size
  - $P_{lap}(unk) = 1 / (N+V)$ same probability for all unks
- how much prob. mass for unks in the above diagram?
  - e.g., $N=10^6$ words, $V=26^{20}$, $V_{obs} = 10^5$, $V_{unk} = 26^{20} - 10^5$

Smoothing: Add Less than One

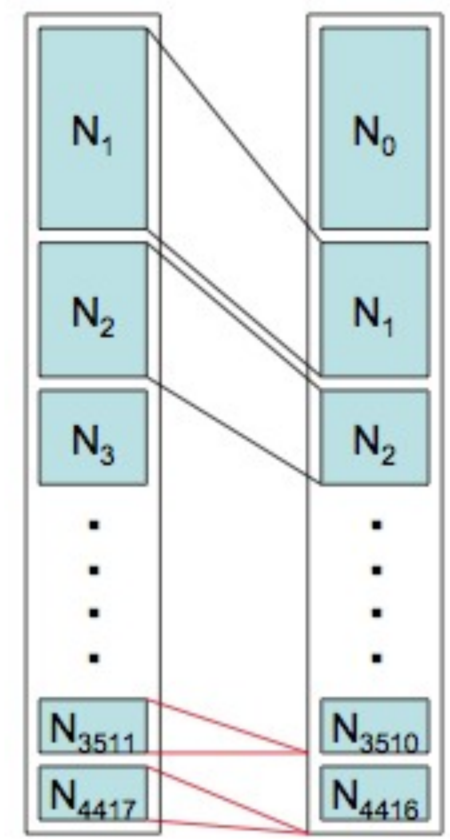
- add one gives too much weight on unseen words!
- solution: add less than one (Lidstone) to each word in V
- $P_{lid}(x) = (c(x) + \lambda) / (N + \lambda V)$ $\quad$ $0<\lambda<1$ is a parameter
  - $P_{lid}(unk) = \lambda / (N+ \lambda V)$ $\quad$ still same for unks, but smaller
- Q: how to tune this λ ? on held-out data!

Smoothing: Witten-Bell

- key idea: use one-count things to guess for zero-counts
  - recurring idea for unknown events, also for Good-Turing
- prob. mass for unseen: T / (N + T)   T: # of seen types
  - 2 kinds of events: one for each token, one for each type
  - = MLE of seeing a new type (T among N+T are new
  - divide this mass evenly among V-T unknown words
- $p_{wb}(x) = T \;/\; (V\text{-}T)(N+T)$ unknown word
  $\quad\quad\quad = c(x) \;/\; (N+T)$ known word
- bigram case more involved; see J&M Chapter for details

Smoothing: Good-Turing

- again, one-count words in training ~ unseen in test
- let N_c = # of words with frequency r in training
- $P_{GT}(x) = c'(x) / N$ where $c'(x) = (c(x)+1)\, N_{c(x)+1} / N_{c(x)}$
- total adjusted mass is $\text{sum}_c\, c'\, N_c = \text{sum}_c\, (c+1)\, N_{c+1} / N$
  - remaining mass: N_1 / N: split evenly among unks



EXAMPLE:

r	Nr	Nr+1	r*	r*/N
0	1000	100	-	1-Z
1	100	40	0.8	Sums
2	40	20	1.5	to
3	20	10	2.0	Z
4	10	6	3.0	
5	6	3	3.0	
⋮	⋮	⋮	⋮	⋮

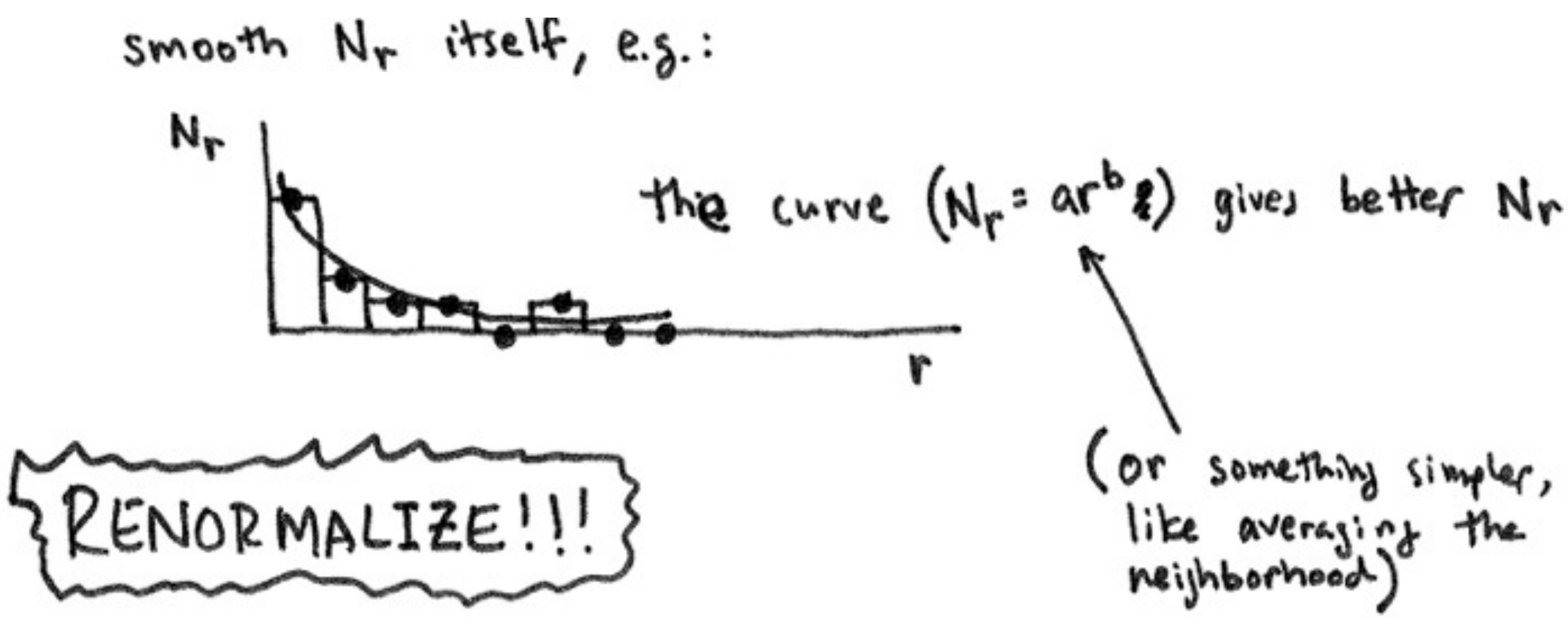
Smoothing: Good-Turing

- from Church and Gale (1991).
  bigram LMs. unigram vocab size = 4×10^5.
  T_r is the frequencies in the held-out data (see $f_{empirical}$).

$r = f_{MLE}$	$f_{empirical}$	f_{Lap}	f_{del}	f_{GT}	N_r	T_r
0	0.000027	0.000137	0.000037	0.000027	74 671 100 000	2 019 187
1	0.448	0.000274	0.396	0.446	2 018 046	903 206
2	1.25	0.000411	1.24	1.26	449 721	564 153
3	2.24	0.000548	2.23	2.24	188 933	424 015
4	3.23	0.000685	3.22	3.24	105 668	341 099
5	4.21	0.000822	4.22	4.22	68 379	287 776
6	5.23	0.000959	5.20	5.19	48 190	251 951
7	6.21	0.00109	6.21	6.21	35 709	221 693
8	7.21	0.00123	7.18	7.24	27 710	199 779
9	8.26	0.00137	8.18	8.25	22 280	183 971

Smoothing: Good-Turing

- Good-Turing is much better than add (less than) one
- problem 1: $N_{cmax+1} = 0$, so $c'max = 0$
  - solution: only adjust counts for those less than k (e.g., 5)
- problem 2: what if $N_c = 0$ for some middle c?
  - solution: smooth N_c itself



Smoothing: Backoff

$$\hat{p}(w_i|w_{i-2}w_{i-1}) = \begin{cases} \tilde{p}(w_i|w_{i-2}w_{i-1}), & \text{if} C(w_{i-2}w_{i-1}w_i) > 0 \\ \alpha_1 p(w_i|w_{i-1}), & \text{if} C(w_{i-2}w_{i-1}w_i) = 0 \\ & \text{and} C(w_{i-1}w_i) > 0 \\ \alpha_2 p(w_i), & \text{otherwise.} \end{cases}$$

Smoothing: Interpolation

$$\begin{aligned}\hat{p}(w_i|w_{i-2}w_{i-1}) &= \lambda_1 p(w_i|w_{i-2}w_{i-1}) \\ &+\lambda_2 p(w_i|w_{i-1}) \\ &+\lambda_3 p(w_i)\end{aligned}$$

subject to the constraint that $\sum_j \lambda_j = 1$

Entropy and Perplexity

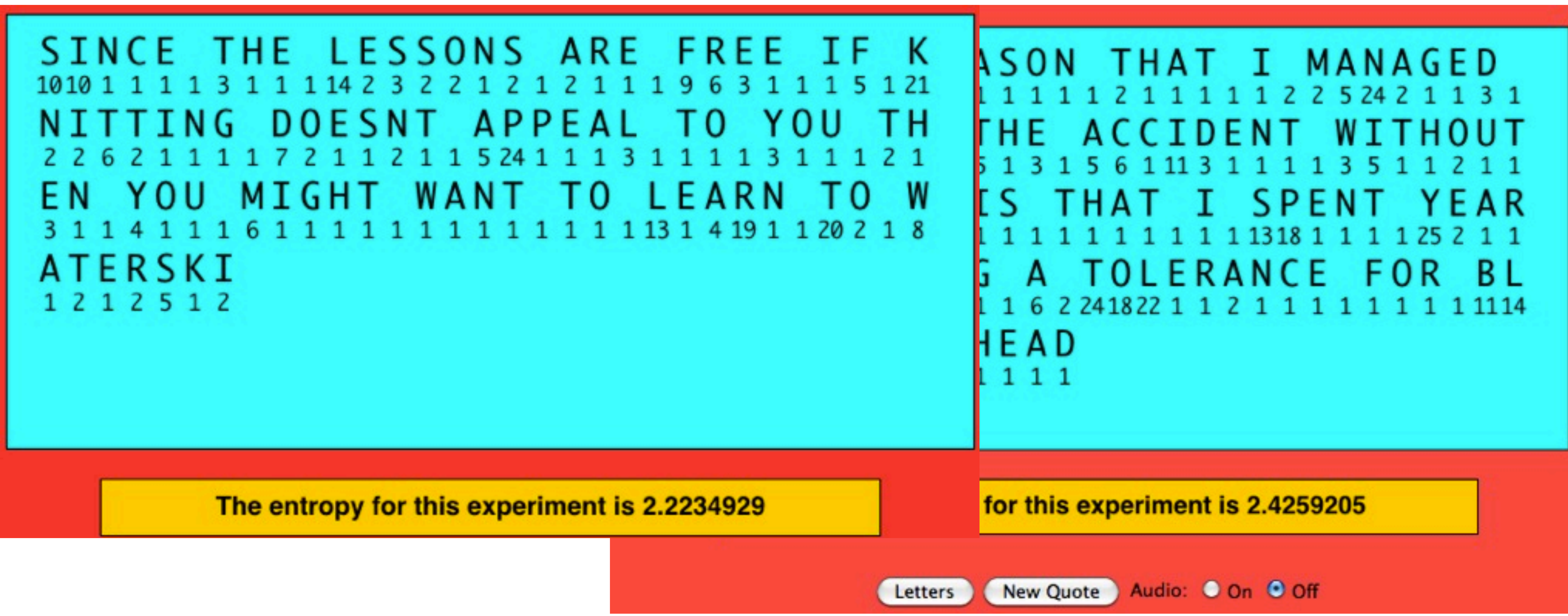
- classical entropy (uncertainty): H(X) = -sum p(x) log p(x)
  - how many "bits" (on average) for encoding
- sequence entropy (distribution over sequences):
  - $H(L) = \lim 1/n\ H(w_1 \dots w_n)$ Q: why 1/n?
  - $= \lim 1/n\ \text{sum\_}\{w \text{ in } L\}\ p(w_1 \dots w_n) \log p(w_1 \dots w_n)$
- Shannon-McMillan-Breiman theorem:
  - $H(L) = \lim -1/n \log p(w_1 \dots w_n)$ don't need all w in L!
  - if w is long enough, just take -1/n log p(w) is enough!
- perplexity is 2^{H(L)}

Perplexity of English

- on 1.5 million WSJ test set:
  - unigram: 962 9.9 bits
  - bigram: 170 7.4 bits
  - trigram: 109 6.8 bits
- higher-order n-grams generally has lower perplexity
  - but higher than trigram is not that significant
- what about human??

Shannon Game

- guess the next letter; compute entropy (bits per char)
- 0-gram: 4.76,   1-gram: 4.03,   2-gram: 3.32,   3-gram: 3.1
- native speaker: ~1.1 (0.6~1.3);   me: ~2.3



http://math.ucsd.edu/~crypto/java/ENTROPY/