## Robotics


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## RoboCup Challenges

- Simulation League
- Small League
- Medium-sized League (less interest)
- SONY Legged League
- Humanoid League


## Small League

- Overhead camera
- Central controlling computer for each team
- Fast and agile
- Winning teams have had the best hardware



## Medium-Sized League

- Largest fully-autonomous robots
- Has been plagued by hardware challenges


## Humanoid League

- Still demonstrating technology and skills (kicking, vision, localization)



## SONY Legged League <br> CMU vs. New South Wales (1999)



CMU vs. New South Wales (2002)


## Special Purpose Vision



Bright Light


Dim Light

## What the Dog Sees



## Making Sense of Sensing

- $\mathrm{P}\left(\right.$ Image $_{\mathrm{t}} \mid$ CameraPose $_{\mathrm{t}}$, World $\left._{\mathrm{t}}\right)$
- P(CameraPose ${ }_{t} \mid$ BodyPose $\left._{t}\right)$
- P(BodyPose ${ }_{t} \mid$ BodyPose $_{t-1}$, Action $\left._{t}\right)$
- $P\left(\right.$ World $_{\mathrm{t}} \mid$ World $\left._{\mathrm{t}-1}\right)$
- $\operatorname{argmax}_{\mathrm{wt}_{t}} \mathrm{P}\left(\mathrm{W}_{\mathrm{t}} \mid \mathrm{I}_{\mathrm{t}}, \mathrm{A}_{\mathrm{t}},\right)=$ $\operatorname{argmax}_{\mathrm{wt}} \sum_{\mathrm{Wt}_{t-1} \mathrm{Ct,Ct-1}, \mathrm{Bt}, \mathrm{Bt}-1} \mathrm{P}\left(\mathrm{l}_{\mathrm{t}} \mid \mathrm{W}_{\mathrm{t}}, \mathrm{C}_{\mathrm{t}}\right)$. $P\left(W_{t} \mid W_{t-1}\right) \cdot P\left(C_{t} \mid B_{t}\right) \cdot P\left(B_{t} \mid B_{t-1}, A_{t}\right)=$
- $\operatorname{argmax}_{w t} \sum_{C_{C} P} P\left(l_{t} \mid W_{t}, C_{t}\right) \cdot \sum_{w t-1} P\left(W_{t} \mid W_{t-}\right.$ 1) $\cdot \sum_{B t} P\left(C_{t} \mid B_{t}\right) \cdot \sum_{B t-1} P\left(B_{t} \mid B_{t-1}, A_{t}\right)$



## The World

- Locations (and orientations and velocities) of
- self
- ball
- other players on same team
- players on other team


## Actions

- Actions can be described at many levels of detail
- low level actions: moving body joints
- intermediate level actions: walking gaits, shooting and passing motions, localization motions, celebration dances
- learned or programmed prior to the game
- higher-level actions: "shoot on goal", "pass to X", "keep away from Y"
- decisions are made at this level during the game


## Choosing Actions to Maximize Utility

- Markov Decision Process
- Set of states X
- Set of actions A
- State transition function: $P\left(X_{t} \mid X_{t-1}, A_{t}\right)$
- Reward function: $R\left(X_{t-1}, A_{t}, X_{t}\right)$
- Discount factor $\gamma$
- Policy: $\pi$ : $\mathrm{X} \mapsto \mathrm{A}$
- maps from states to actions
- Value of a policy:
$-\mathrm{E}\left[\mathrm{R}_{1}+\gamma \mathrm{R}_{2}+\gamma^{2} \mathrm{R}_{3}+\cdots\right]$


## The Reward Function

- Overall Reward function $R(X)$
- reward received when entering state $X$
- example: scoring goal $R=+1$
- example: opponent scores: $\mathrm{R}=-1$
- reward is zero most of the time. We say that reward is "delayed"


## The Value Function

- $\mathrm{V}^{\pi}(\mathrm{X})$ is the expected discounted reward of being in state $X$ and executing policy $\pi$.
- $\mathrm{V}^{\pi}(\mathrm{X})=\sum_{\mathrm{X}^{\prime}} \mathrm{P}\left(\mathrm{X}^{\prime} \mid \mathrm{X}, \pi(\mathrm{X})\right)$.
$\left[\mathrm{R}\left(\mathrm{X}, \pi(\mathrm{X}), \mathrm{X}^{\prime}\right)+\gamma \mathrm{V}^{\pi}\left(\mathrm{X}^{\prime}\right)\right]$


$$
\begin{aligned}
\mathrm{V}^{\pi}(\mathrm{X}) & =0.3 \cdot \gamma(-2)+0.7 \cdot \gamma(1.5) \\
& =0.405
\end{aligned}
$$

## Computing the Optimal Policy by Computing its Value Function

- Let $\mathrm{V}^{*}(\mathrm{X})$ denote the expected discounted reward of following the optimal policy, $\pi^{*}$, starting in state X .
$\mathrm{V}^{*}(\mathrm{X})=\max _{\mathrm{a}} \mathrm{\Sigma}_{\mathrm{X}} \cdot \mathrm{P}\left(\mathrm{X}^{\prime} \mid \mathrm{X}, \mathrm{a}\right)\left[\mathrm{R}\left(\mathrm{X}, \mathrm{a}, \mathrm{X}^{\prime}\right)+\gamma \mathrm{V}^{*}\left(\mathrm{X}^{\prime}\right)\right]$
Value Iteration:
Initialize $V(X)=0$ in all states $X$
repeat until $V$ converges:
for each state $X$, compute $\mathrm{V}^{*}(\mathrm{X}):=\max _{\mathrm{a}} \Sigma_{\mathrm{s}^{\prime}} \mathrm{P}\left(\mathrm{X}^{\prime} \mid \mathrm{X}, \mathrm{a}\right)\left[\mathrm{R}\left(\mathrm{X}, \mathrm{a}, \mathrm{X}^{\prime}\right)+\gamma \mathrm{V}^{*}\left(\mathrm{X}^{\prime}\right)\right]$


## Computing the Optimal Policy

 from $V^{*}$$\pi^{*}(X):=\operatorname{argmax}_{\mathrm{a}} \sum_{\mathrm{X}^{\prime}} \mathrm{P}\left(\mathrm{X}^{\prime} \mid \mathrm{X}, \mathrm{a}\right)\left[\mathrm{R}\left(\mathrm{X}, \mathrm{a}, \mathrm{X}^{\prime}\right)+\gamma \mathrm{V}^{*}\left(\mathrm{X}^{\prime}\right)\right]$

Perform a one-step lookahead, evaluate the resulting states $\mathrm{X}^{\prime}$ using $\mathrm{V}^{*}$, and choose the best action

## Scale-up Problems

- Value Iteration
- Requires $O(|X||A| B)$ time, where $B$ is the branching factor (number of states resulting from an action)
- Not practical for more than 30,000 states
- Not practical for continuous state spaces
- Where do the probability distributions come from?


## Reinforcement Learning

- Learn the transition function and the reward function by experimenting with the environment
- Perform value iteration to compute $\pi^{*}$
- Other methods compute $\mathrm{V}^{*}$ or $\pi^{*}$ directly without learning $P\left(X^{\prime} \mid X, A\right)$ or $R\left(X, A, X^{\prime}\right)$
- Q learning
- SARSA( $\lambda$ )


## Scaling Methods

- Value Function Approximation
- Compact parameterizations of value functions (e.g., as linear, polynomial, or non-linear functions)
- Policy Approximation
- Compact representation of the policy
- Gradient descent in "policy space"


## Multiple Agents

- The Markov Decision Process is a model of only a single agent, but robocup involves multiple cooperative and competitive agents
- There is a separate reward function for each agent, but it depends on the actions of all of the other agents
- $\mathrm{R}_{1}\left(\mathrm{X}, \mathrm{a}_{1}, \ldots, \mathrm{a}_{\mathrm{N}}, \mathrm{X}^{\prime}\right)$
$-R_{2}\left(X, a_{1}, \ldots, a_{N}, X^{\prime}\right)$
$-R_{N}\left(X, a_{1}, \ldots, a_{N}, X^{\prime}\right)$


## Game Theory

- Each agent ("player") has a policy for choosing actions
- The combination of policies results in a value function for each player
- Each player seeks to optimize his/her own value function
- Stable solutions: Nash Equilibrium
- Each player's current policy is a local optimum if all of the other players' policies are kept fixed
- Each player has no incentive to change
- Computing Nash Equilibria in general is a research problem, although there are special cases where solutions are known.


## Stochastic Policies

- In games, the optimal policy may be stochastic (i.e., actions are chosen according to a probability distribution)
$-\pi(\mathrm{X}, \mathrm{A})=$ probability of choosing action A in state X
- Example: Rock, Paper, Scissors
- Nash equilibrium: choose randomly among the three actions


## How to choose actions when you don't know your opponent's policy

- Consider one or more policies that your opponent is likely to play
- Design a policy that works well against all of them


## The Segway League?



## Non-Mobile Robot Motion Planning

- Industrial robot arms
- Degrees of Freedom (one for each independent direction in which a robot or one of its effectors can move)
How many (internal) degrees of freedom does this arm have?



## Kinematics and Dynamics

- Kinematic State
- joint angle of each joint
- Dynamic State
- Kinematic State + velocities and accelerations of each joint


## Holonomic vs. Non-Holonomic

- Automobile (on a plane)
-3 degrees of freedom ( $x, y, \theta$ )
- only 2 controllable degrees of freedom
- wheels and steering
- Holonomic: number of degrees of freedom = number of controllable degrees of freedom
- easier to control, often more expensive
- Non-Holonomic: degrees of freedom > controllable degrees of freedom


## Path Planning

- Want to move robot arm from one location (conf-1) to another (conf-2)



## Two Different Coordinate Systems

- Locations can be specified in two different coordinate systems
- Workspace Coordinates
- position of end-effector ( $x, y, z$ ) and possibly its orientation (roll, pitch,yaw)
- Joint Coordinates
- angle of each joint



## Forward and Inverse Kinematics

- Forward Kinematics
- Given joint angles compute workspace coordinates - easy
- Inverse Kinematics
- Given workspace coordinates compute joint angles
- hard: may exist multiple solutions (often infinitely many)
- Path planning involves
- finding a path
- easy to do in joint angle space
- avoiding obstacles
- easy to do in workspace


## Computing Obstacle Representations in C-Space

- Must convert each obstacle from a region of workspace to a region in configuration space
- Often done by sampling
- generate grid of points in C-space
- test if corresponding point is occupied by obstacle
- Interesting computational geometry challenge


# Path Planning in Configuration Space 

- Cell Decomposition Methods
- Potential Field Methods
- Voronoi Graph Methods
- Probabilistic Roadmap Methods
- key problem: C-Space is continuous!


## Cell Decomposition

- Define a grid of cells for free space
- A path consists of a sequence of cells
- Legal moves: go from center of one cell to center of 8 neighboring cells:

- Converts path planning to discrete search problem (use $\mathrm{A}^{*}$ or Value Iteration)


## Cell Decomposition

cell color indicates optimal value function (distance to goal along optimal policy)


## Problems with Cell Decomposition

- How do we handle cells that overlap obstacles?
- ignore: algorithm is incomplete (possible plan will not be found)
- include: algorithm is unsound (plan may not work)
- Number of cells grows exponentially with number of joints (dimensionality of C-Space)
- Paths may touch (or pass too close to) obstacles


## Solutions to Cell Problems

- Cells too big/too small
- Use variable resolution cell size. Degree cell size near obstacles
- Cell scaling
- Voronoi and Roadmap methods
- Touching obstacles
- Potential Field Methods


## Potential Field Method

- Define a "cost" for getting close to obstacles ("the potential")
- Find optimal path that minimizes the combined path length + cost



## Potential Field Result



## Voronoi Methods ("skeletonization")

- Define set of points equidistant from two or more obstacles
- This has lower dimensionality (often 1D). Finitely-many intersections.
- Path: from start to Voronoi skeleton, along skeleton, from skeleton to end



## Problems with Voronoi Method

- Resulting paths maximize "clearance" from obstacles
- Does not work well in large open spaces
- Path goes through middle of space
- Computing the diagram can be difficult in C-Space.


## Probabilistic Roadmap

- Draw a sample of points in C-Space.
- Keep those points that are in free space.
- Compute Delauney Triangulation of the sample points
- This gives a graph of points in free space
- Search in this graph


## Probabilistic Roadmap



## Scaling Problems

- All of these methods do not scale to very high dimensional spaces
- Probabilistic roadmap and Voronoi method scale best
- Probabilistic roadmap is cheapest to compute
- Sampling can be dynamically refined based on initial paths


## Executing Robot Plans

- Path only specifies the kinematic state of the robot arm
- Actually moving the arm must deal with dynamics: acceleration, mass, friction, etc.
- Control theory has well-developed methods for smoothly following a trajectory
- e.g., PID controllers (proportional integral derivative controllers)


## Robotics Summary

- Robots live in partially-observable, stochastic environments that may contain other cooperative and competitive agents
- Robot Tasks
- localization
- mapping
- action selection
- planning (single agent; multiple cooperative agents; multiple competitive agents; teams)
- action execution


## Robot Planning

- For single-agent stochastic environments
- MDP model
- Value Iteration
- Reinforcement Learning
- For multiple-agent stochastic environments
- Game theory model
- Still a research topic
- For single-agent deterministic (non-mobile) environment
- Configuration space planning

