A Time-Delay Jitter-Insensitive Continuous-Time Bandpass $\Delta\Sigma$ Modulator Architecture

Anurag Pulincherry, Michael Hufford, Eric Naviasky, and Un-Ku Moon

Abstract—In this paper, we present a new continuous-time bandpass delta–sigma ($\Delta\Sigma$) modulator architecture with mixer inside the feedback loop. The proposed bandpass $\Delta\Sigma$ modulator is insensitive to time-delay jitter in the digital-to-analog conversion feedback pulse, unlike conventional continuous-time bandpass $\Delta\Sigma$ modulators. The sampling frequency of the proposed $\Delta\Sigma$ modulator can be less than the center frequency of the input narrow-band signal.

Index Terms—Bandpass, continuous time, delta–sigma ($\Delta\Sigma$), frequency translating, time-delay jitter.

I. INTRODUCTION

BANDPASS delta–sigma ($\Delta\Sigma$) modulators are used to digitize high-frequency narrow-band signals. They are widely used in digital IF receivers for digitizing IF signals [1]–[3]. In a typical digital IF receiver, RF signal is down converted to a low IF frequency. Continuous-time bandpass $\Delta\Sigma$ modulators are used to digitize low IF signals because they can be designed at very low power. The loop filter is implemented using simple RC or Gm–C integrators, which are easy to implement.

Implementation of continuous-time bandpass $\Delta\Sigma$ modulators at low IF frequency is a viable solution for analog-to-digital conversion (DAC) in radio receivers. However, this is not true for converters operating at high IF frequency. At high IF frequency [4], [5], the performance of continuous-time bandpass $\Delta\Sigma$ modulators is limited by time-delay jitter and pulse width jitter in the DAC feedback waveform [6]. Continuous-time bandpass $\Delta\Sigma$ modulators operating at high IF frequency also requires high-Q resonators, accurately tuned to the IF frequency.

Continuous-time bandpass $\Delta\Sigma$ modulators operating at high IF frequency can be realized by introducing frequency translation inside the $\Delta\Sigma$ loop [5]. This class of continuous-time bandpass $\Delta\Sigma$ modulators are called frequency-translating $\Delta\Sigma$ modulators. A frequency-translating $\Delta\Sigma$ modulator consist of a low-Q wide-band bandpass resonator followed by a down-conversion mixer and a low-pass $\Delta\Sigma$ modulator. The output of the low-pass $\Delta\Sigma$ modulator is upconverted and fed back to the bandpass resonator to complete the feedback loop. Most of the
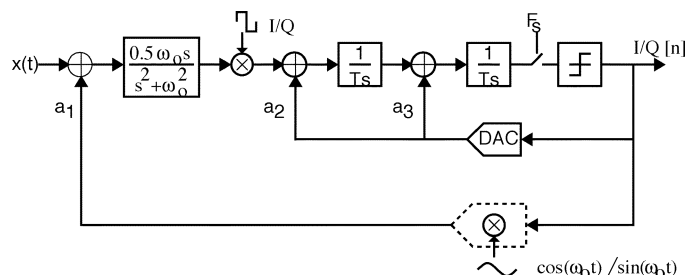
Fig. 1. Proposed architecture.

loop gain comes from the integrators in the low-pass $\Delta\Sigma$ modulator. Therefore, high-Q resonators are not needed. The design requirements on the mixer and opamps in the $\Delta\Sigma$ loop are relaxed.

In the frequency-translating $\Delta\Sigma$ modulator described in [5], the low-pass $\Delta\Sigma$ modulator in the frequency-translating loop is implemented by switched capacitor circuits, sampled at four times the IF frequency. This architecture can be modified by implementing the low-pass $\Delta\Sigma$ modulator in continuous time and sampling at the IF frequency. The resulting frequency-translating $\Delta\Sigma$ modulator has the interesting property that it is insensitive to time-delay jitter in DAC feedback pulse. In this paper, we present a design methodology for the system-level design of the modified frequency-translating $\Delta\Sigma$ modulator architecture. The output spectrum and the time-delay jitter insensitivity of the architecture are verified through behavioral simulations in MATLAB.

II. PROPOSED ARCHITECTURE

The proposed frequency-translating bandpass $\Delta\Sigma$ modulator architecture is shown in Fig. 1. The input narrow-band signal is amplified by a low-Q wide-band resonator. The resonance frequency of the bandpass resonator is the IF frequency. The output of the bandpass resonator is downconverted to baseband or a low IF frequency by a mixer in the $\Delta\Sigma$ loop. The resulting signal is digitized by a single-bit continuous-time low-pass or low IF $\Delta\Sigma$ modulator.

The down-converted signal has a baseband or low IF portion and a high-frequency portion centered around twice the IF frequency. The antialias filtering inherent in the continuous-time $\Delta\Sigma$ modulator will filter away the high-frequency component in the down-converted narrow-band signal. The bandwidth of the down-converted signal is usually much smaller than the IF frequency. Hence, the sampling frequency can be equal to or less than the IF frequency. Note that, in conventional continuous-time bandpass $\Delta\Sigma$ modulators, the sampling frequency is usually four times the IF frequency.
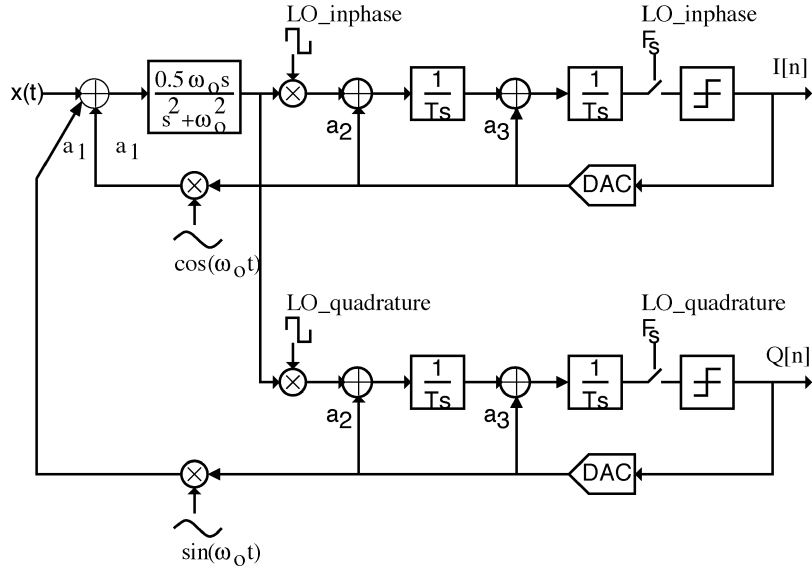
Fig. 2.   Direct-conversion frequency-translating $\Delta\Sigma$ architecture.
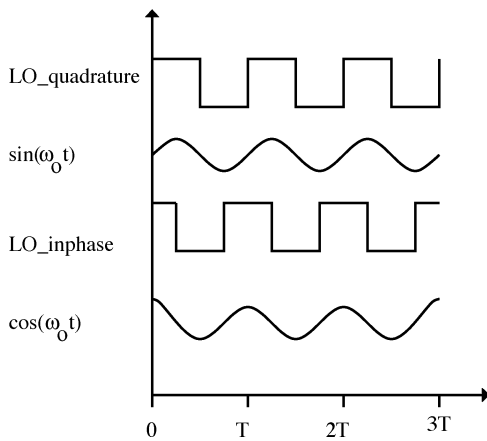
Fig. 3.   Modulating in-phase and quadrature signals.

Fig. 4.   Typical DAC feedback pulse.

The output of the single-bit continuous-time low-pass or low-IF $\Delta\Sigma$ modulator is upconverted to the IF frequency. If the digitized output of the low-pass or the low-IF $\Delta\Sigma$ modulator is a digital "1," a sinusoid pulse in phase with the modulating signal or the local oscillator (LO) signal is fed back to the bandpass resonator. If the quantized output is a digital "0," a sinusoid pulse which is 180° out of phase with the LO is used as the DAC feedback pulse. We could use a square pulse or any other shape for DAC feedback. The $\Delta\Sigma$ modulator coefficients a_1, a_2, and a_3 would change appropriately.

If the input narrow-band signal is down-converted to baseband, the resulting in-phase (I) signal and the quadrature (Q) signal have to be digitized separately, just like in a direct-conversion radio receiver [3], [5]. The architecture of the direct-conversion frequency-translating $\Delta\Sigma$ modulator is shown in Fig. 2. The LO signal used in the down-conversion and up-conversion mixers in the $\Delta\Sigma$ loops is shown in Fig. 3. A typical DAC feedback waveform is given in Fig. 4. In this paper, we confine our analysis to direct-conversion frequency-translating $\Delta\Sigma$ modulator architecture.
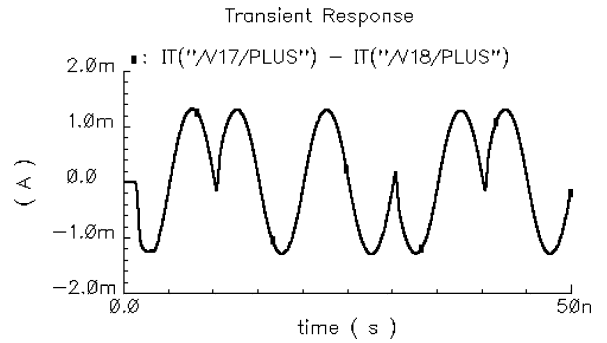
III. OPEN-LOOP RESPONSE OF FREQUENCY-TRANSLATING $\Delta\Sigma$ MODULATOR

The loop filter design of $\Delta\Sigma$ modulators is usually done in the z domain. The discrete-time loop transfer function is then mapped to an equivalent continuous-time transfer function using pulse-invariant transformation [7]. This means that the sampled, open-loop pulse response of the continuous-time $\Delta\Sigma$ modulator is identical to the open-loop impulse response of the discrete-time $\Delta\Sigma$ modulator.

An analogous mapping of an open-loop response is possible between a continuous-time $\Delta\Sigma$ modulator and a continuous-time frequency-translating $\Delta\Sigma$ modulator. Let us consider just the quadrature path of the direct-conversion $\Delta\Sigma$ modulator shown in Fig. 2 for simplicity. In order to find the open-loop response of any feedback system, we first break the feedback loop and feed in a test input to observe the open-loop response.

Let us break the feedback loop of the frequency-translating $\Delta\Sigma$ modulator at the input of the up-conversion mixer. The resulting structure is a cascade of an up-conversion mixer, bandpass resonator, down-conversion mixer followed by the continuous-time low-pass $\Delta\Sigma$ modulator. The $\Delta\Sigma$ modulator low-pass filters the mixer output due to its inherent antialiasing property. Let us ignore the low-pass $\Delta\Sigma$ modulator for simplicity.
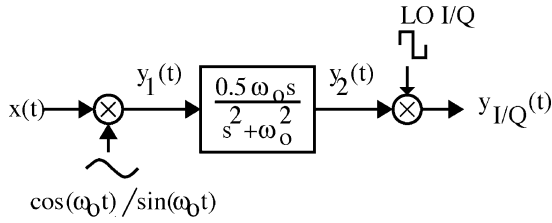
Fig. 5. Open-loop system.

The resulting open-loop system is shown in Fig. 5. The test input $x(t)$ is a nonreturn-to-zero (NRZ) pulse.

The transfer function of the bandpass resonator is given by (1), where ω_o is the IF frequency in radians per second. The modulating signal in the down-conversion mixer is a square wave. The gain of the down-conversion mixer is given by

$$H_{\mathrm{op}}(s) = \frac{0.5\omega_o s}{s^2 + \omega_o^2}. \tag{1}$$

$$G_{\mathrm{mixer}} = \frac{2}{\pi}. \tag{2}$$

For input frequencies very close to the resonance frequency of the bandpass resonator, the transfer function given by (1) can be approximated to

$$H\left(j(\omega + \delta\omega)\right) = 0.5\frac{j(\omega_o + \delta\omega)\omega_o}{\omega_o^2 - (\omega_o + \delta\omega)^2} \simeq \frac{\omega_o}{j4\delta\omega}. \tag{3}$$

The transfer function given by (3) can be combined with the mixer gain to get the overall transfer function at the output of the down-conversion mixer, given by

$$H_{\mathrm{eq}}(s = j\omega) = G_{\mathrm{mixer}}\frac{\omega_o}{j4\delta\omega} = \frac{f_o}{s} \tag{4}$$

where f_o is the IF frequency.

It is clear that the open-loop cascade of the up-conversion mixer, bandpass resonator, and down-conversion mixer can be approximated to a simple integrator as far as the test input is concerned. The transient response of the open-loop system shown in Fig. 5 for NRZ pulse input is given in Fig. 6. The dotted response in the same figure is the NRZ pulse response of the equivalent integrator given by (4). The two responses match very well.

IV. DERIVATION OF THE FEEDBACK COEFFICIENTS OF THE FREQUENCY-TRANSLATING $\Delta\Sigma$ MODULATOR

The open-loop pulse response of the cascade of the up-conversion mixer, bandpass resonator, and down-conversion mixer shown in Fig. 5 is identical to that of an integrator given by (4). We can replace the cascade of the up-conversion mixer, bandpass resonator, and down-conversion mixer, with the equivalent integrator to obtain a third-order continuous-time low-pass $\Delta\Sigma$ modulator, shown in Fig. 7. Designing a continuous-time low-pass $\Delta\Sigma$ modulator is a well-understood subject area. The feedback coefficients are given by $a_1 = -0.2$, $a_2 = -0.3$, and $a_3 = -0.6416$, for an IF frequency of 100 MHz, sampled at 100 MHz.
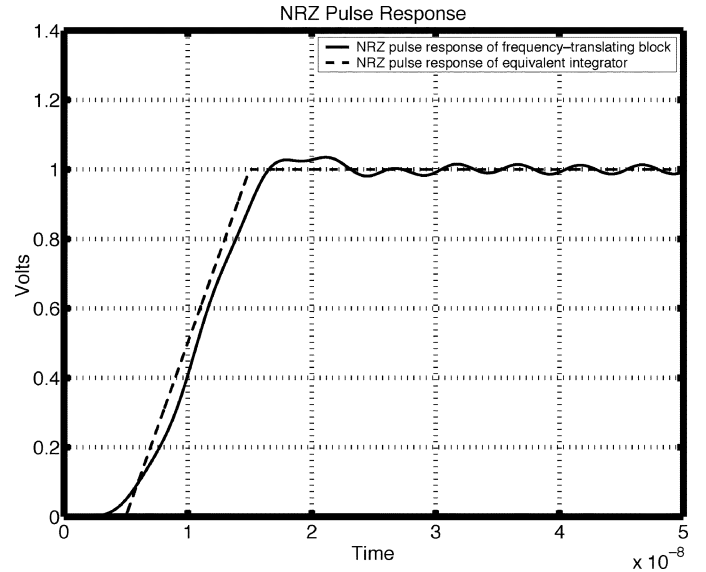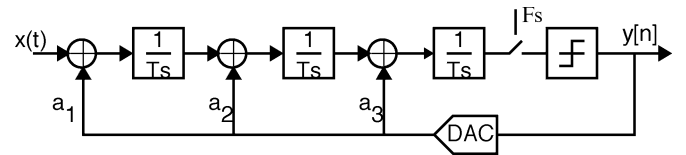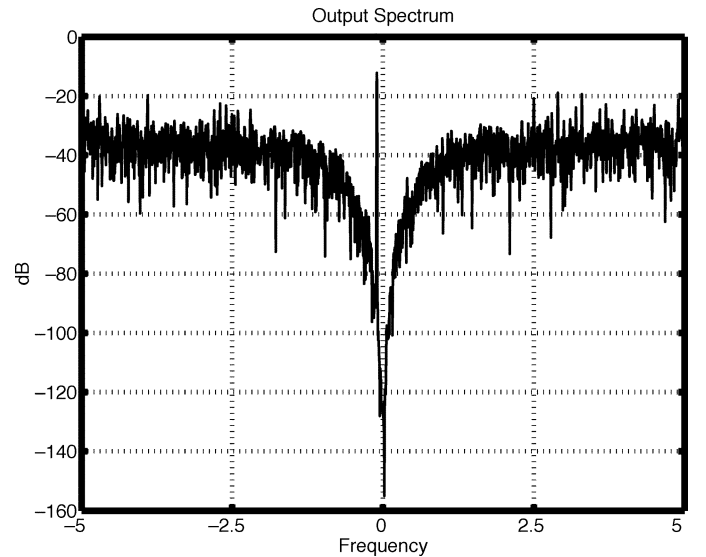

Fig. 6. Open-loop response.

Fig. 7. Equivalent third-order continuous-time $\Delta\Sigma$ modulator.

Fig. 8. Complex spectrum of the proposed frequency-translating $\Delta\Sigma$ modulator.

The frequency-translating $\Delta\Sigma$ modulator architecture shown in Fig. 2, can be simulated using state space equations. The complex output spectrum of the frequency-translating $\Delta\Sigma$ modulator shown in Fig. 2 is given in Fig. 8. The simulated SNR is 96 dB for 200-kHz bandwidth. The DAC feedback signal in the frequency-translating $\Delta\Sigma$ modulator analyzed was an NRZ pulse modulated by a sinusoid. It is possible to design a frequency-translating $\Delta\Sigma$ modulator with a modulated return-to-zero (RZ) pulse as the DAC feedback. The feedback coefficients will change appropriately.
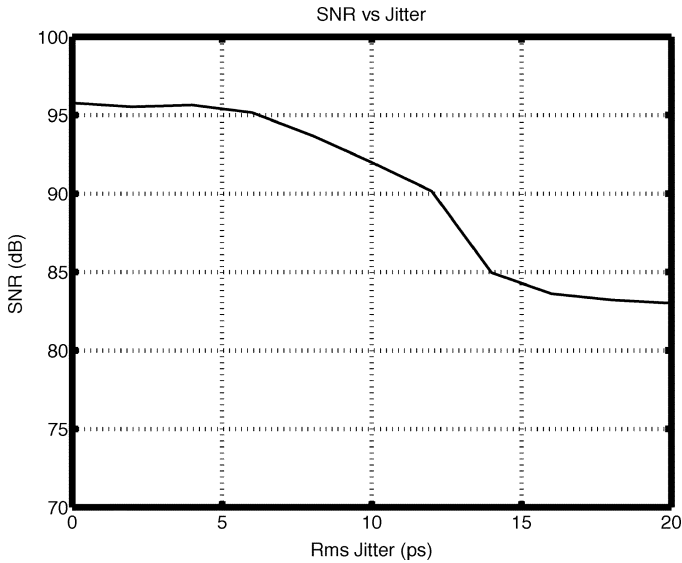
Fig. 9. SNR versus rms time-delay jitter plot of the proposed frequency-translating $\Delta\Sigma$ modulator.

Fig. 10. SNR versus rms time-delay jitter plot of a conventional continuous-time bandpass $\Delta\Sigma$ modulator.

V. TIME-DELAY JITTER INSENSITIVITY OF A FREQUENCY-TRANSLATING $\Delta\Sigma$ MODULATOR

It is well known that continuous-time low-pass $\Delta\Sigma$ modulators with RZ DAC feedback are insensitive to time-delay jitter [8]. This is because the open-loop output of the first integrator in the loop filter of a continuous-time $\Delta\Sigma$ modulator is proportional to the area under the DAC feedback pulse. The area under the DAC feedback pulse is invariant under time-delay jitter.

In Section III, we saw that the open-loop response of the cascade of up-conversion mixer, bandpass resonator, and down-conversion mixer is identical to that of an integrator. The feedback coefficient of the proposed modulator was obtained in Section IV by mapping the frequency-translating modulator in to an equivalent continuous-time low-pass $\Delta\Sigma$ modulator. Thus, the open-loop response or loop dynamics of the proposed frequency-translating $\Delta\Sigma$ modulator is the same as that of a continuous-time low-pass $\Delta\Sigma$ modulator. Therefore, a continuous-time frequency-translating $\Delta\Sigma$ modulator is insensitive to DAC feedback time-delay jitter similar to a continuous-time low-pass $\Delta\Sigma$ modulator.

In order to verify the time-delay jitter insensitivity of a frequency-translating $\Delta\Sigma$ modulator, we design a modulator with an IF frequency of 200 MHz and sampled at 100 MHz. If we use one cycle of sinusoid pulse per sampling clock period, for DAC feedback, then it corresponds to an upconverted RZ DAC pulse. A plot of simulated SNR versus time-delay jitter for such a $\Delta\Sigma$ modulator is given in Fig. 9. Note that conventional continuous-time bandpass $\Delta\Sigma$ modulators are sensitive to time-delay jitter in DAC feedback pulse. A plot of simulated SNR versus time-delay jitter for a conventional continuous-time bandpass $\Delta\Sigma$ modulator at 200 MHz is shown in Fig. 10.

As shown in the figures, it is clear that the SNR of the proposed modulator does not degrade even for a large rms time-delay jitter.
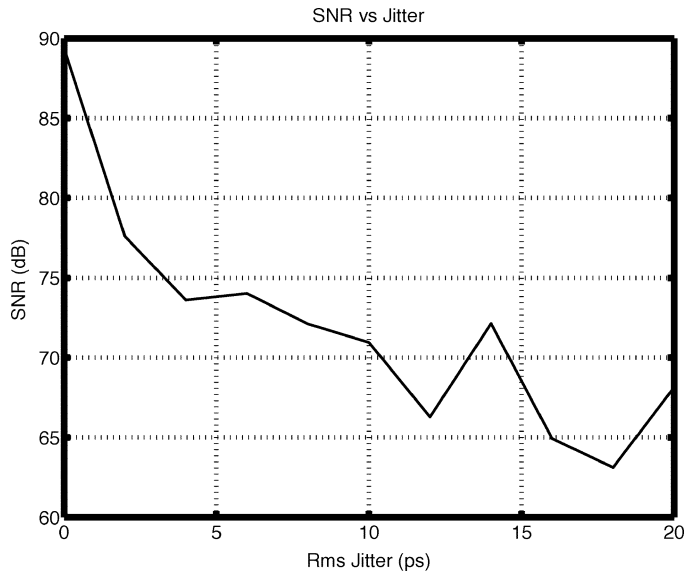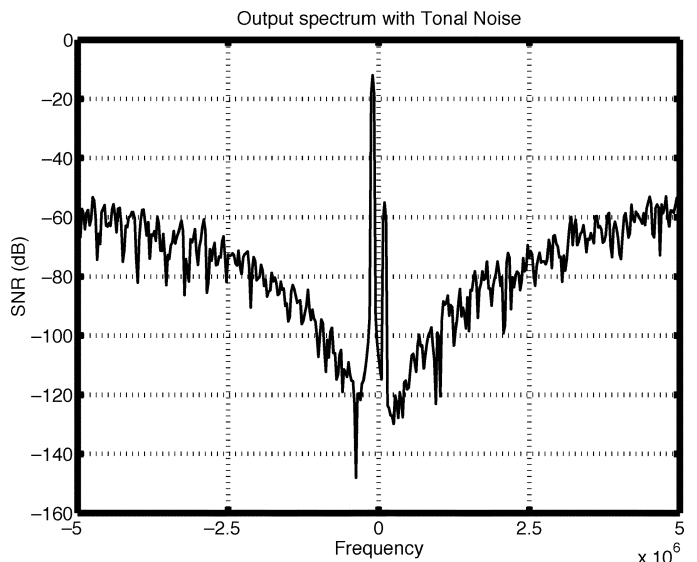

Fig. 11. Complex-spectrum with tonal noise added to sinusoid DAC pulse.

VI. CHOICE OF DAC FEEDBACK PULSE SHAPE

In Section V, we saw that the proposed frequency-translating $\Delta\Sigma$ modulator is insensitive to time-delay jitter. However, it is sensitive to phase noise in the sinusoid pulse used for DAC feedback. This is analogous to a continuous-time low-pass $\Delta\Sigma$ modulator being sensitive to pulse-width jitter in the DAC feedback pulse, although it is insensitive to time-delay jitter. The complex spectrum of the frequency-translating $\Delta\Sigma$ modulator, when tonal noise is added to the feedback sinusoid pulse, at an offset frequency, is shown in Fig. 11.

The phase-modulated noise tone appears in the signal band at a frequency offset from the IF signal. However, the noise floor does not increase. The tonal noise in the sinusoid pulse is modulated by the signal as well as the quantization noise. However, the quantization noise is attenuated in the signal band. Although the quantization noise outside the band of interest is high, the

phase noise is attenuated away from the carrier frequency. Thus, the phase noise is modulated significantly by the input signal only and appears as a "skirt" around the signal.

It is well known that continuous-time $\Delta\Sigma$ modulators with switched-capacitor DAC feedback are insensitive to pulse-width jitter and time-delay jitter in the DAC feedback pulse [1]. This because the DAC feedback waveform in a continuous-time low-pass $\Delta\Sigma$ modulator with switched-capacitor DAC is an exponentially decaying pulse instead of a rectangular pulse. Thus, pulse-width variation has a reduced effect on the area under the DAC feedback pulse. The SNR degradation due to jitter in frequency-translating $\Delta\Sigma$ modulators can be eliminated by giving the sinusoid pulse an exponentially decaying envelope. Such a pulse can be easily generated by switching an appropriately damped *LC* tank circuit to the reference voltages, depending on the quantizer output of the low-pass $\Delta\Sigma$ modulator in the frequency-translating loop. The analysis and behavioral simulation of such a frequency-translating $\Delta\Sigma$ modulator is beyond the scope of this paper.

VII. TIME INVARIANCE AND FREQUENCY-TRANSLATING $\Delta\Sigma$ MODULATOR

A frequency-translating $\Delta\Sigma$ modulator uses mixers in the $\Delta\Sigma$ modulator loop. Ideal mixers are linear but not time-invariant. Hence, conventional stability criteria based on location of closed-loop poles [9] may not be applicable to a frequency-translating $\Delta\Sigma$ modulator.

However, the DAC output in a frequency-translating $\Delta\Sigma$ modulator is updated once in every sampling clock period. Time invariance can be achieved if we ensure that the loop response of the frequency-translating $\Delta\Sigma$ modulator is time-shifted appropriately, when the input RZ or NRZ DAC pulse to the upconversion mixer is time-shifted by one sampling clock period. It is easy to show that this is true for the open-loop system

shown in Fig. 5. This class of systems are called periodic linear time-invariant systems [5].

VIII. CONCLUSION

We introduced a modified frequency-translating $\Delta\Sigma$ modulator architecture for digitizing narrow-band signals at high IF frequency. The proposed modulator can be mapped to an equivalent continuous-time low-pass $\Delta\Sigma$ modulator. The proposed modulator is insensitive to time-delay jitter in DAC feedback pulse, similar to continuous-time low-pass $\Delta\Sigma$ modulators.

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